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Citation: Trinh, Luan, Vo, Thuc, Thai, Huu-Tai and Mantari, J. L. (2017) Size-dependent behaviour of functionally graded sandwich microplates under mechanical and thermal loads. Composites Part B: Engineering, 124. pp. 218-241. ISSN 1359-8368

Published by: Elsevier

URL: <https://doi.org/10.1016/j.compositesb.2017.05.042>  
<<https://doi.org/10.1016/j.compositesb.2017.05.042>>

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# Size-dependent behaviour of functionally graded sandwich microplates under mechanical and thermal loads

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## Abstract

This paper presents the static bending, free vibration and buckling behaviours of functionally graded sandwich microplates under mechanical and thermal loads. Governing equations of both higher-order shear deformation and quasi-3D theories are derived based on the variational principle and modified couple stress theory. Apart from mechanical load, the temperature profiles considered are either uniform or linear distribution through the thickness, which results in changes of material properties and stress resultants. Numerical results are obtained using Navier solutions. The difference between quasi-3D and 2D models in dealing with mechanical and thermal load is discussed. Temperature-dependent and temperature-independent material properties are examined. The effects of geometry and power-law index together with mechanical loads and various temperature distributions on the size-dependent behaviours of functionally graded sandwich plates are also investigated.

*Keywords:* Functionally graded plate, sandwich plate, modified couple stress theory, quasi-3D theory.

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## 1. Introduction

Functionally graded materials (FGMs) are a kind of composite materials in which the material components are the mixture of two or more constituents. By gradually changing the volume fraction of materials, the desirable material properties can be tailored continuously, and thus avoiding the delamination phenomenon occurred in laminated composites. In addition, being usually manufactured by metal and ceramic components, which are qualified for superior strength and thermal insulation, functionally graded (FG) structures possess striking features for engineering applications. The use of FGMs becomes more promising when they are combined with sandwich structures to create FG sandwich structures. In such structures, a FG layer is usually sandwiched between homogeneous skins to form FG core-homogeneous skins sandwich; otherwise, a homogeneous core is inserted to two FG layers to create homogeneous core- FG skins sandwich. By this way, the structures can be customised with thicker skins or symmetric configurations.

Applications of small-scale structures inspire new research on the behaviours of micro/nano structures. There are three main methods to analyse such small-scale structures including molecular dynamics simulation, hybrid molecular-continuum methods, and non-classical continuum methods [1]. Among them, the third one is computationally effective and thus widely applicable to solid structures. The development of these micro-continuum methods can be outlined from the introduction of the additional degrees of freedom of rotation at each material particle in Cosserat continua [2] by introducing material length scale parameters. Insightful discussions about classical couple stress theories [3-6] and other non-classical continuum theories including strain gradient [3, 4], modified strain gradient [5], nonlocal elasticity [6] and modified nonlocal elasticity [7-11] can be found in [5, 10, 12-14]. The main challenge of these theories is how to determine correctly material length scale parameters. The Modified Couple Stress Theory (MCST) has advantages over other size-dependent theories since it has only one material length scale parameter and includes symmetric couple stress. It is the reason why this theory is widely used in practice. Following is the brief review of the MCST which was developed based on the classical plate theory (CPT), first-order shear deformation theory (FSDT) and higher-order shear deformation theory (HSDT). It should be noted that CPT model is only appropriate for thin plates due to neglecting shear deformation effect. Based on this model, Ke et al. [15] investigated the static bending, free vibration and buckling behaviours of FG annular microplates with various boundary conditions (BCs). The formulations for arbitrary shape and free vibration of FG rectangular microplates was examined by Asghari and Taati [16]. Geometric nonlinearity was accounted for in Taati's work [17] for buckling and post-buckling behaviours of FG microplates under different BCs using analytical solutions. To overcome the limitation of the CPT model, FSDT model has been proposed by assuming the in-plane

displacements vary linearly through the thickness. Thus, a shear correction factor is necessary. Using this model, analytical solutions were developed to linear and nonlinear bending, vibration and buckling analysis of simply supported FG microplates by Thai and Choi [18]. Jung et al. [19, 20] included the elastic foundation in the behaviour of FG microplates. Ansari et al. [21, 22] also developed a nonlinear model for the vibration, bending and post-buckling analysis of FG microplates. In order to eliminate the use of the shear correction factor and obtain a better prediction of responses for thick plates, several HSDTs have been proposed. These models were applied to investigate bending and free vibration behaviours of FG microplates by Thai and Kim [23] using analytical solutions and annular/circular microplates by Eshraghi [24] using the differential quadrature (DQ) method. Thai and Vo [25] developed a MCST sinusoidal theory for FG microplates and derived analytical solutions for deflections and natural frequencies of simply supported microplates. He et al. [26] presented a MCST four-variable refined plate model using analytical solution for the FG microplates. Lou et al. [27] then developed this model for a unified framework including the von Karman's geometric nonlinearity. It should be noted that above studies [26-30], the normal strain or normal deformation, which becomes significant for thick plates, is not included. In order to take into account both shear and normal deformation effects, Nguyen et al. [28] presented four-variable quasi-3D model to analyse the bending, vibration and buckling behaviours of FG microplates using the isogeometric solutions.

Thermal effect also has been analysed for FG micro structures in some publications. Reddy and Kim [29] developed theoretical formulation for mechanical and thermal analysis with a general nonlinear model containing cubic and quadratic variations of the in-plane and transverse displacements for FG microplates. They also derived the linear analytical solutions of bending, vibration and buckling behaviours for this model under mechanical loads [30]. Mirsalehi et al. [31] investigated the stability of thin FG microplates under mechanical and thermal loads using CPT based on the spline finite strip method. Using CPT, Ashoori and Vanini [32] also studied thermal buckling of annular FG microplates resting on an elastic medium accounting for geometrically nonlinear effect and snap-through behaviour. Utilising the DQ method, Eshaghi et al. [33] analysed static bending and free vibration responses of FG annular/circular plates based on CPT, FSDT and HSDT models.

Based on the MCST, this paper presents the quasi-3D models, which include both shear and stretching effects, for the bending, vibration and buckling behaviours of FG sandwich microplates under mechanical and thermal loads. The refined plate model, which is obtained by degenerating the thickness stretching effect, is applied to figure out the difference between the 2D and quasi-3D solutions. The unified temperature profile is applied to describe the uniform and linear distribution through the thickness. The effects of geometry and power-law index together with mechanical loads and various temperature distributions on the size-dependent behaviours of FG sandwich microplates are also

investigated.

## 2. Theoretical formulation

### 2.1. Functionally graded sandwich plate and temperature-dependent material properties

In this paper, FG plate and two types of FG sandwich plates are considered. The material properties including Young's modulus  $E(z)$ , thermal expansion coefficients  $\alpha(z)$  and mass density  $\rho(z)$  are expressed using the rule of mixture:

$$P(z, T) = [P_c(z, T) - P_m(z, T)]V_c(z) + P_m(z, T) \quad (1)$$

where  $P(z)$  is either  $E(z)$ ,  $\alpha(z)$  or  $\rho(z)$ ;  $c$  and  $m$  indicate ceramic and metal. Material properties in the temperature-dependent analysis are calculated for ceramic and metal as [34]:

$$P(T) = P_0 (P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \quad (2)$$

where  $P_{-1}, P_1, P_2$  and  $P_3$ , given in Table 2, are the temperature-dependent coefficients. To simplify the numerical calculation, Poisson's ratio is measured at the middle-plane's temperature for both metal and ceramic.

The volume fraction of ceramic is described by the power law as below:

*a. FG plate:*

$$V_c(z) = \left( \frac{z}{h} + \frac{1}{2} \right)^p \quad (3a)$$

*b. Sandwich plate with FG core – homogeneous skins:*

$$V_c(z) = \begin{cases} 0 & \text{if } z \in [h_0, h_1] \\ \left( \frac{z - h_1}{h_2 - h_1} \right)^p & \text{if } z \in [h_1, h_2] \\ 1 & \text{if } z \in [h_2, h_3] \end{cases} \quad (3b)$$

*c. Sandwich plate with ceramic core – FG skins:*

$$V_c(z) = \begin{cases} \left( \frac{z - h_0}{h_1 - h_0} \right)^p & \text{if } z \in [h_0, h_1] \\ 1 & \text{if } z \in [h_1, h_2] \\ \left( \frac{z - h_3}{h_2 - h_3} \right)^p & \text{if } z \in [h_2, h_3] \end{cases} \quad (3c)$$

*d. Sandwich plate with metal core – FG skins:*

$$V_c(z) = \begin{cases} 1 - \left( \frac{z - h_0}{h_1 - h_0} \right)^p & \text{if } z \in [h_0, h_1] \\ 0 & \text{if } z \in [h_1, h_2] \\ 1 - \left( \frac{z - h_3}{h_2 - h_3} \right)^p & \text{if } z \in [h_2, h_3] \end{cases} \quad (3d)$$

where  $p$  is the power law index.

Here, uniform and linear temperatures through the thickness is considered and described by:

$$T(x, y, z) = 300 + \Delta T_1(x, y) + \frac{z}{h} \Delta T_2(x, y) \quad (4)$$

where  $\Delta T_1$  and  $\Delta T_2$  determine the uniform increase and the gradient of temperature through the thickness, respectively. In this paper, unless otherwise stated,  $\Delta T_2 = 100K$  which leads to  $100K$  higher temperature at the top surface compared to the bottom one.

## 2.2. Kinematics and constitutive relations

Consider a FG sandwich plate with the coordinate and cross-section shown in Fig. 1. By applying the modified couple stress theory, the variation of strain energy is related to both strain and curvature tensors as [12]:

$$\delta \Pi = \int_V (\boldsymbol{\sigma} \delta \boldsymbol{\varepsilon} + \mathbf{m} \delta \boldsymbol{\chi}) dV \quad (5)$$

where  $\boldsymbol{\varepsilon}$  and  $\boldsymbol{\chi}$  are the strain and symmetric curvature tensor defined by:

$$\boldsymbol{\varepsilon} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \quad (6a)$$

$$\boldsymbol{\chi} = \frac{1}{2} (\nabla \boldsymbol{\theta} + (\nabla \boldsymbol{\theta})^T) \quad (6b)$$

$\boldsymbol{\sigma}$  and  $\mathbf{m}$  are the corresponding stress and deviatoric part of the symmetric couple stress tensors defined by:

$$\boldsymbol{\sigma} = \lambda \text{tr}(\boldsymbol{\varepsilon}) \mathbf{I} + 2\mu \boldsymbol{\varepsilon} \quad (7a)$$

$$\mathbf{m} = 2l^2 \mu \boldsymbol{\chi} \quad (7b)$$

in which,  $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$  and  $\mu = \frac{E}{2(1+\nu)}$  are Lamé's constants,  $l$  is the material length scale parameter [12],  $\mathbf{u} = (u_1, u_2, u_3)$  and  $\boldsymbol{\theta} = (\theta_x, \theta_y, \theta_z)$  are the displacement and rotation vectors expressed below.

The displacement field which includes the normal stretching effect is assumed to an arbitrary point as:

$$u_1(x, y, z, t) = U(x, y, t) - z \frac{\partial W_b(x, y, t)}{\partial x} - f(z) \frac{\partial W_s(x, y, t)}{\partial x} \quad (8a)$$

$$u_2(x, y, z, t) = V(x, y, t) - z \frac{\partial W_b(x, y, t)}{\partial y} - f(z) \frac{\partial W_s(x, y, t)}{\partial y} \quad (8b)$$

$$u_3(x, y, z, t) = W_b(x, y, t) + W_s(x, y, t) + g(z) W_z(x, y, t) \quad (8c)$$

where  $U$  and  $V$  are in-plane displacements,  $W_b, W_s$  and  $W_z$  are bending, shear and stretching displacements of a point on the middle plane.  $f(z)$  and  $g(z) = 1 - \frac{df(z)}{dz}$  are the shape functions which distribute the effect of  $W_s$  and  $W_z$  across the thickness. Two sets of shape functions are applied [35-37]:

$$\text{Third-order shear deformation theory (TSDT): } f(z) = \frac{4}{3} \frac{z^3}{h^2} \quad (9a)$$

$$\text{Sinusoidal shear deformation theory (SSDT): } f(z) = z - \frac{h}{\pi} \sin \frac{\pi z}{h} \quad (9b)$$

The rotation vector is expressed as:

$$\boldsymbol{\theta} = \frac{1}{2} \text{curl} \mathbf{u} = \frac{1}{2} \left[ \left( \frac{\partial u_3}{\partial y} - \frac{\partial u_2}{\partial z} \right) \mathbf{e}_x + \left( \frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x} \right) \mathbf{e}_y + \left( \frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y} \right) \mathbf{e}_z \right] \quad (10a)$$

$$\theta_x = \frac{1}{2} \text{curl} u|_{\mathbf{e}_x} = \frac{\partial W_b}{\partial y} + \frac{1}{2} \left( 1 + \frac{\partial f(z)}{\partial z} \right) \frac{\partial W_s}{\partial y} + \frac{1}{2} g(z) \frac{\partial W_z}{\partial y} \quad (10b)$$

$$\theta_y = \frac{1}{2} \text{curl} u|_{\mathbf{e}_y} = -\frac{\partial W_b}{\partial x} - \frac{1}{2} \left( 1 + \frac{\partial f(z)}{\partial z} \right) \frac{\partial W_s}{\partial x} - \frac{1}{2} g(z) \frac{\partial W_z}{\partial x} \quad (10c)$$

$$\theta_z = \frac{1}{2} \text{curl} u|_{\mathbf{e}_z} = \frac{1}{2} \left( \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \right) \quad (10d)$$

The strain components related to above displacement field are presented by substituting Eq. (8) to Eq. (6a):

$$\varepsilon_{xx} = \frac{\partial U}{\partial x} - z \frac{\partial^2 W_b}{\partial x^2} - f(z) \frac{\partial^2 W_s}{\partial x^2} - \alpha(z, T) \Delta T(x, y, z) \quad (11a)$$

$$\varepsilon_{yy} = \frac{\partial V}{\partial y} - z \frac{\partial^2 W_b}{\partial y^2} - f(z) \frac{\partial^2 W_s}{\partial y^2} - \alpha(z, T) \Delta T(x, y, z) \quad (11b)$$

$$\varepsilon_{zz} = \frac{\partial g(z)}{\partial z} W_z - \alpha(z, T) \Delta T(x, y, z) \quad (11c)$$

$$\gamma_{xy} = 2\varepsilon_{xy} = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} - 2z \frac{\partial^2 W_b}{\partial x \partial y} - 2f(z) \frac{\partial^2 W_s}{\partial x \partial y} \quad (11d)$$

$$\gamma_{xz} = 2\varepsilon_{xz} = g(z) \left( \frac{\partial W_s}{\partial x} + \frac{\partial W_z}{\partial x} \right) \quad (11e)$$

$$\gamma_{yz} = 2\varepsilon_{yz} = g(z) \left( \frac{\partial W_s}{\partial y} + \frac{\partial W_z}{\partial y} \right) \quad (11f)$$

and the curvature tensor is given by substituting Eq. (10) to Eq. (6b):

$$\chi_{xx} = \frac{\partial^2 W_b}{\partial y \partial x} + \frac{1}{2} \left( 1 + \frac{\partial f(z)}{\partial z} \right) \frac{\partial^2 W_s}{\partial y \partial x} + \frac{1}{2} g(z) \frac{\partial^2 W_z}{\partial y \partial x} \quad (12a)$$

$$\chi_{yy} = -\frac{\partial^2 W_b}{\partial x \partial y} - \frac{1}{2} \left( 1 + \frac{\partial f(z)}{\partial z} \right) \frac{\partial^2 W_s}{\partial x \partial y} - \frac{1}{2} g(z) \frac{\partial^2 W_z}{\partial x \partial y} \quad (12b)$$

$$\chi_{zz} = 0 \quad (12c)$$

$$\chi_{xy} = \frac{1}{2} \left[ \frac{\partial^2 W_b}{\partial y^2} - \frac{\partial^2 W_b}{\partial x^2} + \frac{1}{2} \left( 1 + \frac{\partial f(z)}{\partial z} \right) \left( \frac{\partial^2 W_s}{\partial y^2} - \frac{\partial^2 W_s}{\partial x^2} \right) + \frac{1}{2} g(z) \left( \frac{\partial^2 W_z}{\partial y^2} - \frac{\partial^2 W_z}{\partial x^2} \right) \right] \quad (12d)$$

$$\chi_{xz} = \frac{1}{4} \left[ \frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 U}{\partial x \partial y} + \frac{\partial^2 f(z)}{\partial z^2} \frac{\partial W_s}{\partial y} + \frac{\partial g(z)}{\partial z} \frac{\partial W_z}{\partial y} \right] \quad (12e)$$

$$\chi_{yz} = \frac{1}{4} \left[ \frac{\partial^2 V}{\partial x \partial y} - \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 f(z)}{\partial z^2} \frac{\partial W_s}{\partial x} - \frac{\partial g(z)}{\partial z} \frac{\partial W_z}{\partial x} \right] \quad (12f)$$

Substituting Eqs. (11) and (12) to Eq. (7), the thermal strain, stress and deviatoric part of couple stress tensors are obtained, respectively:

$$\left\{ \begin{matrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{xy} \end{matrix} \right\} = \left[ \begin{matrix} Q_{11} & Q_{12} & Q_{12} & 0 & 0 & 0 \\ & Q_{22} & Q_{23} & 0 & 0 & 0 \\ & & Q_{33} & 0 & 0 & 0 \\ & & & Q_{44} & 0 & 0 \\ & & & & Q_{55} & 0 \\ & sym. & & & & Q_{66} \end{matrix} \right] \left\{ \begin{matrix} \frac{\partial U}{\partial x} - z \frac{\partial^2 W_b}{\partial x^2} - f(z) \frac{\partial^2 W_s}{\partial x^2} - \alpha(z, T) \Delta T(x, y, z) \\ \frac{\partial V}{\partial y} - z \frac{\partial^2 W_b}{\partial y^2} - f(z) \frac{\partial^2 W_s}{\partial y^2} - \alpha(z, T) \Delta T(x, y, z) \\ \frac{\partial g(z)}{\partial z} W_z - \alpha(z, T) \Delta T(x, y, z) \\ g(z) \left( \frac{\partial W_s}{\partial x} + \frac{\partial W_z}{\partial x} \right) \\ g(z) \left( \frac{\partial W_s}{\partial y} + \frac{\partial W_z}{\partial y} \right) \\ \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} - 2z \frac{\partial^2 W_b}{\partial x \partial y} - 2f(z) \frac{\partial^2 W_s}{\partial x \partial y} \end{matrix} \right\} \quad (13)$$



$$\left\{ \begin{matrix} m_{xx} \\ m_{yy} \\ m_{zz} \\ m_{xz} \\ m_{yz} \\ m_{xy} \end{matrix} \right\} = 2l^2 \mu \left\{ \begin{matrix} \frac{\partial^2 W_b}{\partial y \partial x} + \frac{1}{2} \left( 1 + \frac{\partial f(z)}{\partial z} \right) \frac{\partial^2 W_s}{\partial y \partial x} + \frac{1}{2} g(z) \frac{\partial^2 W_z}{\partial y \partial x} \\ - \frac{\partial^2 W_b}{\partial x \partial y} - \frac{1}{2} \left( 1 + \frac{\partial f(z)}{\partial z} \right) \frac{\partial^2 W_s}{\partial x \partial y} - \frac{1}{2} g(z) \frac{\partial^2 W_z}{\partial x \partial y} \\ 0 \\ \frac{1}{4} \left( \frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 U}{\partial x \partial y} + \frac{\partial^2 f(z)}{\partial z^2} \frac{\partial W_s}{\partial y} + \frac{\partial g(z)}{\partial z} \frac{\partial W_z}{\partial y} \right) \\ \frac{1}{4} \left( \frac{\partial^2 V}{\partial x \partial y} - \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 f(z)}{\partial z^2} \frac{\partial W_s}{\partial x} - \frac{\partial g(z)}{\partial z} \frac{\partial W_z}{\partial x} \right) \\ \frac{1}{2} \left[ \frac{\partial^2 W_b}{\partial y^2} - \frac{\partial^2 W_b}{\partial x^2} + \frac{1}{2} \left( 1 + \frac{\partial f(z)}{\partial z} \right) \left( \frac{\partial^2 W_s}{\partial y^2} - \frac{\partial^2 W_s}{\partial x^2} \right) + \frac{1}{2} g(z) \left( \frac{\partial^2 W_z}{\partial y^2} - \frac{\partial^2 W_z}{\partial x^2} \right) \right] \end{matrix} \right\} \quad (14)$$

where

$$[Q_{ii}, Q_{ij}] = [Q_{ii}^{2D}, Q_{ij}^{2D}] = \begin{cases} \left[ \frac{E(z)}{1-\nu(z)^2}, \frac{E(z)\nu(z)}{1-\nu(z)^2} \right] (i, j = \overline{1, 3}) \\ \left[ \frac{E(z)}{2[1+\nu(z)]}, 0 \right] (i, j = \overline{4, 6}) \end{cases} \quad \text{for 2D model} \quad (15a)$$

$$[Q_{ii}, Q_{ij}] = [Q_{ii}^{3D}, Q_{ij}^{3D}] = \begin{cases} \left[ \frac{E(z)[1-\nu(z)]}{[1+\nu(z)][1-2\nu(z)]}, \frac{E(z)\nu(z)}{[1+\nu(z)][1-2\nu(z)]} \right] (i, j = \overline{1, 3}) \\ \left[ \frac{E(z)}{2[1+\nu(z)]}, 0 \right] (i, j = \overline{4, 6}) \end{cases} \quad \text{for quasi-3D model.} \quad (15b)$$

### 2.3. Variational formulation

Hamilton's principle is applied to obtain the equations of motion:

$$\int_{t_1}^{t_2} (\delta \Pi + \delta V + \delta K) dt \quad (16)$$

where  $\delta \Pi$ ,  $\delta K$  and  $\delta V$  denote the variation of strain, kinetic energy and work done by external forces.

The variation of strain energy is rewritten in terms of mid-plane displacements as:

$$\delta \Pi = \int_{A-h/2}^{h/2} (\sigma_{ij} \delta \varepsilon_{ij} + m_{ij} \delta \chi_{ij}) dz dA$$

$$\begin{aligned}
&= \int_{A-h/2}^{h/2} \int \left[ (\sigma_{xx} \delta \varepsilon_{xx} + \sigma_{yy} \delta \varepsilon_{yy} + \sigma_{zz} \delta \varepsilon_{zz} + \sigma_{xz} \delta \gamma_{xz} + \sigma_{yz} \delta \gamma_{yz} + \sigma_{xy} \delta \gamma_{xy}) \right. \\
&\quad \left. + (m_{xx} \delta \chi_{xx} + m_{yy} \delta \chi_{yy} + m_{zz} \delta \chi_{zz} + 2m_{xz} \delta \chi_{xz} + 2m_{yz} \delta \chi_{yz} + 2m_{xy} \delta \chi_{xy}) \right] dz dA \\
&= \int_A \left[ \left( N_{xx} \frac{\partial \delta U}{\partial x} - M_{xx} \frac{\partial^2 \delta W_b}{\partial x^2} - P_{xx} \frac{\partial^2 \delta W_s}{\partial x^2} \right) + \left( N_{yy} \frac{\partial \delta V}{\partial y} - M_{yy} \frac{\partial^2 \delta W_b}{\partial y^2} - P_{yy} \frac{\partial^2 \delta W_s}{\partial y^2} \right) + O_{zz} \delta W_z \right. \\
&\quad + Q_{xz} \left( \frac{\partial \delta W_s}{\partial x} + \frac{\partial \delta W_z}{\partial x} \right) + Q_{yz} \left( \frac{\partial \delta W_s}{\partial y} + \frac{\partial \delta W_z}{\partial y} \right) + N_{xy} \left( \frac{\partial \delta U}{\partial y} + \frac{\partial \delta V}{\partial x} \right) - 2M_{xy} \frac{\partial^2 \delta W_b}{\partial x \partial y} - 2P_{xy} \frac{\partial^2 \delta W_s}{\partial x \partial y} \\
&\quad + R_{xx} \left( \frac{\partial^2 \delta W_b}{\partial x \partial y} + \frac{1}{2} \frac{\partial^2 \delta W_s}{\partial x \partial y} \right) + \frac{1}{2} (S_{xx} - S_{yy}) \frac{\partial^2 \delta W_s}{\partial x \partial y} + \frac{1}{2} (T_{xx} - T_{yy}) \frac{\partial^2 \delta W_z}{\partial x \partial y} - R_{yy} \left( \frac{\partial^2 \delta W_b}{\partial x \partial y} + \frac{1}{2} \frac{\partial^2 \delta W_s}{\partial x \partial y} \right) \\
&\quad + \frac{1}{2} R_{xz} \left( \frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 U}{\partial x \partial y} \right) + \frac{1}{2} R_{yz} \left( \frac{\partial^2 V}{\partial x \partial y} - \frac{\partial^2 U}{\partial y^2} \right) + \frac{1}{2} X_{xz} \left( \frac{\partial W_s}{\partial y} + \frac{\partial W_z}{\partial y} \right) - \frac{1}{2} X_{yz} \left( \frac{\partial W_s}{\partial x} + \frac{\partial W_z}{\partial x} \right) \\
&\quad \left. + \frac{1}{2} R_{xy} \left( \frac{\partial^2 W_b}{\partial y^2} - \frac{\partial^2 W_b}{\partial x^2} \right) + \frac{1}{2} R_{xy} \left( \frac{\partial^2 W_s}{\partial y^2} - \frac{\partial^2 W_s}{\partial x^2} \right) + \frac{1}{2} S_{xy} \left( \frac{\partial^2 W_s}{\partial y^2} - \frac{\partial^2 W_s}{\partial x^2} \right) + \frac{1}{2} T_{xy} \left( \frac{\partial^2 W_z}{\partial y^2} - \frac{\partial^2 W_z}{\partial x^2} \right) \right] dA \quad (17)
\end{aligned}$$

where the stress resultants are expressed as:

$$(N_{ij}, M_{ij}, P_{ij}, Q_{ij}, O_{ij}) = \int_{-h/2}^{h/2} \left( 1, z, f(z), g(z), \frac{\partial g(z)}{\partial z} \right) \sigma_{ij} dz - (N_{ij}^T, M_{ij}^T, P_{ij}^T, 0, O_{ij}^T) \quad (18a)$$

$$(R_{ij}, S_{ij}, T_{ij}, X_{ij}) = \int_{-h/2}^{h/2} \left( 1, \frac{\partial f(z)}{\partial z}, g(z), \frac{\partial^2 f(z)}{\partial z^2} \right) m_{ij} dz \quad (18b)$$

where

$$(N_{ij}^T, M_{ij}^T, P_{ij}^T, O_{ij}^T) = \int_{-h/2}^{h/2} \left( 1, z, f(z), \frac{\partial g(z)}{\partial z} \right) (Q_{ii} + 2Q_{ij}) \alpha(z, T) \Delta T(z) dz, (i, j = \overline{1, 3}) \quad (19)$$

It is noticeable that the integration is written for FG plate only, for sandwich plate cumulative

formulation is applied, i.e.  $\int_{-h/2}^{h/2} \mathbb{F}(z) dz = \int_{h_0}^{h_1} \mathbb{F}(z) dz + \int_{h_1}^{h_2} \mathbb{F}(z) dz + \int_{h_2}^{h_3} \mathbb{F}(z) dz$  where  $\mathbb{F}(z)$  is an arbitrary

function of  $z$ .

Substituting Eqs. (13), (14) and (19) into Eq. (18), the stress resultants can be described in terms of mid-plane displacements as in the Appendix.

The variation of the work done by transverse load  $q$  and in-plane load  $P_0$  is presented as:

$$\delta V = - \int_A \left[ P_0 \left[ \delta W_b' (W_b' + W_s') + \delta W_s' (W_b' + W_s') \right] + q (\delta W_b + \delta W_s + g \delta W_z) \right] dA \quad (20)$$

The variation of kinetic energy is presented by:

$$\begin{aligned}
\delta K &= \int_{A-h/2}^{h/2} \rho(z) (\dot{u}_1 \delta \dot{u}_1 + \dot{u}_2 \delta \dot{u}_2 + \dot{u}_3 \delta \dot{u}_3) dz dA \\
&= \int_A \left\{ I_0 \left[ \dot{U} \delta \dot{U} + \dot{V} \delta \dot{V} + (\dot{W}_b + \dot{W}_s) \delta (\dot{W}_b + \dot{W}_s) \right] + J_0 \left[ (\dot{W}_b + \dot{W}_s) \delta \dot{W}_z + \dot{W}_z \delta (\dot{W}_b + \dot{W}_s) \right] \right. \\
&\quad - I_1 \left( \dot{U} \frac{\partial \delta \dot{W}_b}{\partial x} + \frac{\partial \dot{W}_s}{\partial x} \delta \dot{U} + \dot{V} \frac{\partial \delta \dot{W}_b}{\partial y} + \frac{\partial \dot{W}_b}{\partial y} \delta \dot{V} \right) + I_2 \left( \frac{\partial \dot{W}_b}{\partial x} \frac{\partial \delta \dot{W}_b}{\partial x} + \frac{\partial \dot{W}_b}{\partial y} \frac{\partial \delta \dot{W}_b}{\partial y} \right) \\
&\quad - J_1 \left( \dot{U} \frac{\partial \delta \dot{W}_s}{\partial x} + \frac{\partial \dot{W}_s}{\partial x} \delta \dot{U} + \dot{V} \frac{\partial \delta \dot{W}_s}{\partial y} + \frac{\partial \dot{W}_s}{\partial y} \delta \dot{V} \right) + K_2 \left( \frac{\partial \dot{W}_s}{\partial x} \frac{\partial \delta \dot{W}_s}{\partial x} + \frac{\partial \dot{W}_s}{\partial y} \frac{\partial \delta \dot{W}_s}{\partial y} \right) \\
&\quad \left. + J_2 \left( \frac{\partial \dot{W}_b}{\partial x} \frac{\partial \delta \dot{W}_s}{\partial x} + \frac{\partial \dot{W}_s}{\partial x} \frac{\partial \delta \dot{W}_b}{\partial x} + \frac{\partial \dot{W}_b}{\partial y} \frac{\partial \delta \dot{W}_s}{\partial y} + \frac{\partial \dot{W}_s}{\partial y} \frac{\partial \delta \dot{W}_b}{\partial y} \right) + K_0 \dot{W}_z \delta \dot{W}_z \right\} dA
\end{aligned} \tag{21}$$

where

$$(I_0, I_1, I_2, J_0, J_1, J_2, K_0, K_2) = \int_{-h/2}^{h/2} (1, z, z^2, g(z), f(z), zf(z), g^2(z), f^2(z)) \rho(z) dz \tag{22}$$

Substituting Eqs. (17), (20) and (21) into Eq. (16), integrating by parts and gathering the coefficients of  $\delta U, \delta V, \delta W_b, \delta W_s$  and  $\delta W_z$ , the equations of motion can be obtained:

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} + \frac{1}{2} \frac{\partial^2 R_{xz}}{\partial x \partial y} + \frac{1}{2} \frac{\partial^2 R_{yz}}{\partial y^2} = I_0 \ddot{U} - I_1 \frac{\partial \ddot{W}_b}{\partial x} - J_1 \frac{\partial \ddot{W}_s}{\partial x} \tag{23a}$$

$$\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} - \frac{1}{2} \frac{\partial^2 R_{xz}}{\partial x^2} - \frac{1}{2} \frac{\partial^2 R_{yz}}{\partial x \partial y} = I_0 \ddot{V} - I_1 \frac{\partial \ddot{W}_b}{\partial y} - J_1 \frac{\partial \ddot{W}_s}{\partial y} \tag{23b}$$

$$\begin{aligned}
&\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} - \frac{\partial^2 R_{xx}}{\partial x \partial y} + \frac{\partial^2 R_{yy}}{\partial x \partial y} - \frac{\partial^2 R_{xy}}{\partial y^2} + \frac{\partial^2 R_{xy}}{\partial x^2} + P_0 + q \\
&= I_0 (\ddot{W}_b + \ddot{W}_s) + J_0 \ddot{\phi}_z + I_1 \left( \frac{\partial \ddot{U}}{\partial x} + \frac{\partial \ddot{V}}{\partial y} \right) - I_2 \nabla^2 \ddot{W}_b - J_2 \nabla^2 \ddot{W}_s
\end{aligned} \tag{23c}$$

$$\begin{aligned}
&\frac{\partial^2 P_{xx}}{\partial x^2} + \frac{\partial^2 P_{yy}}{\partial y^2} + \frac{\partial Q_{yz}}{\partial y} + \frac{\partial Q_{xz}}{\partial x} + 2 \frac{\partial^2 P_{xy}}{\partial x \partial y} - \frac{1}{2} \frac{\partial^2 R_{xx}}{\partial x \partial y} - \frac{1}{2} \frac{\partial^2 S_{xx}}{\partial x \partial y} + \frac{1}{2} \frac{\partial^2 R_{yy}}{\partial x \partial y} + \frac{1}{2} \frac{\partial^2 S_{yy}}{\partial x \partial y} \\
&+ \frac{1}{2} \frac{\partial X_{xz}}{\partial y} - \frac{1}{2} \frac{\partial X_{yz}}{\partial x} - \frac{1}{2} \frac{\partial^2 R_{xy}}{\partial y^2} - \frac{1}{2} \frac{\partial^2 S_{xy}}{\partial y^2} + \frac{1}{2} \frac{\partial^2 R_{xy}}{\partial x^2} + \frac{1}{2} \frac{\partial^2 S_{xy}}{\partial x^2} + P_0 + q \\
&= I_0 (\ddot{W}_b + \ddot{W}_s) + J_0 \ddot{\phi}_z + J_1 \left( \frac{\partial \ddot{U}}{\partial x} + \frac{\partial \ddot{V}}{\partial y} \right) - J_2 \nabla^2 \ddot{W}_b - K_2 \nabla^2 \ddot{W}_s
\end{aligned} \tag{23d}$$

$$\begin{aligned}
&-O_{zz} + \frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} - \frac{1}{2} \frac{\partial^2 T_{xx}}{\partial x \partial y} + \frac{1}{2} \frac{\partial^2 T_{yy}}{\partial x \partial y} - \frac{1}{2} \frac{\partial X_{xz}}{\partial y} + \frac{1}{2} \frac{\partial X_{yz}}{\partial x} - \frac{1}{2} \frac{\partial^2 T_{xy}}{\partial y^2} + \frac{1}{2} \frac{\partial^2 T_{xy}}{\partial x^2} + gq \\
&= J_0 (\ddot{W}_b + \ddot{W}_s) + K_0 \ddot{\phi}_z
\end{aligned} \tag{23e}$$

where  $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ ,  $P_0 = P_x^0 \frac{\partial^2 (W_b + W_s)}{\partial x^2} + P_y^0 \frac{\partial^2 (W_b + W_s)}{\partial y^2} + P_{xy}^0 \frac{\partial^2 (W_b + W_s)}{\partial x \partial y}$

in which  $P_x^0, P_y^0$  and  $P_{xy}^0$  are the in-plane mechanical/thermal equivalent forces (in vibration and buckling analysis).

The governing equations are expressed in terms of displacements as:

$$\begin{aligned} & A_{11} \frac{\partial^2 U}{\partial x^2} + A_{66} \frac{\partial^2 U}{\partial y^2} - \frac{1}{4} A_m \left( \frac{\partial^4 U}{\partial x^2 \partial y^2} + \frac{\partial^4 U}{\partial y^4} \right) + (A_{12} + A_{66}) \frac{\partial^2 V}{\partial x \partial y} + \frac{1}{4} A_m \left( \frac{\partial^4 V}{\partial x^3 \partial y} + \frac{\partial^4 V}{\partial x \partial y^3} \right) - B_{11} \frac{\partial^3 W_b}{\partial x^3} \\ & - (B_{12} + 2B_{66}) \frac{\partial^3 W_b}{\partial x \partial y^2} - B_{11}^s \frac{\partial^3 W_s}{\partial x^3} - (B_{12}^s + 2B_{66}^s) \frac{\partial^3 W_s}{\partial x \partial y^2} + K_{13} \frac{\partial \varphi_z}{\partial x} - \frac{\partial N_{xx}^T}{\partial x} = I_0 \ddot{U} - I_1 \frac{\partial \ddot{W}_b}{\partial x} - J_1 \frac{\partial \ddot{W}_s}{\partial x} \end{aligned} \quad (24a)$$

$$\begin{aligned} & A_{22} \frac{\partial^2 V}{\partial y^2} + A_{66} \frac{\partial^2 V}{\partial x^2} - \frac{1}{4} A_m \left( \frac{\partial^4 V}{\partial x^4} + \frac{\partial^4 V}{\partial x^2 \partial y^2} \right) + (A_{12} + A_{66}) \frac{\partial^2 U}{\partial x \partial y} + \frac{1}{4} A_m \left( \frac{\partial^4 U}{\partial x^3 \partial y} + \frac{\partial^4 U}{\partial x \partial y^3} \right) - B_{22} \frac{\partial^3 W_b}{\partial y^3} \\ & - (B_{12} + 2B_{66}) \frac{\partial^3 W_b}{\partial x^2 \partial y} - B_{22}^s \frac{\partial^3 W_s}{\partial y^3} - (B_{12}^s + 2B_{66}^s) \frac{\partial^3 W_s}{\partial x^2 \partial y} + K_{23} \frac{\partial \varphi_z}{\partial y} - \frac{\partial N_{yy}^T}{\partial y} = I_0 \ddot{V} - I_1 \frac{\partial \ddot{W}_b}{\partial y} - J_1 \frac{\partial \ddot{W}_s}{\partial y} \end{aligned} \quad (24b)$$

$$\begin{aligned} & B_{11} \frac{\partial^3 U}{\partial x^3} + (B_{12} + 2B_{66}) \frac{\partial^3 U}{\partial x \partial y^2} + (B_{12} + 2B_{66}) \frac{\partial^3 V}{\partial x^2 \partial y} + B_{22} \frac{\partial^3 V}{\partial y^3} - (D_{11} + A_m) \frac{\partial^4 W_b}{\partial x^4} \\ & - (2D_{12} + 4D_{66} + 2A_m) \frac{\partial^4 W_b}{\partial x^2 \partial y^2} - (D_{22} + A_m) \frac{\partial^4 W_b}{\partial y^4} - \left[ D_{11}^s + \frac{1}{2} (A_m + B_m) \right] \frac{\partial^4 W_s}{\partial x^4} \\ & - (2D_{12}^s + 4D_{66}^s) \frac{\partial^4 W_s}{\partial x^2 \partial y^2} - \left[ D_{22}^s + \frac{1}{2} (A_m + B_m) \right] \frac{\partial^4 W_s}{\partial y^4} + L_{13} \frac{\partial^2 \varphi_z}{\partial x^2} \\ & + L_{23} \frac{\partial^2 \varphi_z}{\partial y^2} - \frac{1}{2} E_m \frac{\partial^4 \varphi_z}{\partial x^4} - \frac{1}{2} E_m \frac{\partial^4 \varphi_z}{\partial y^4} + P_0 + q - \frac{\partial^2 M_{xx}^T}{\partial x^2} - \frac{\partial^2 M_{yy}^T}{\partial y^2} \\ & = I_0 (\ddot{W}_b + \ddot{W}_s) + I_1 \left( \frac{\partial \ddot{U}}{\partial x} + \frac{\partial \ddot{V}}{\partial y} \right) - I_2 \nabla^2 \ddot{W}_b - J_2 \nabla^2 \ddot{W}_s + J_0 \ddot{\varphi}_z \end{aligned} \quad (24c)$$

$$\begin{aligned} & B_{11}^s \frac{\partial^3 U}{\partial x^3} + (B_{12}^s + 2B_{66}^s) \frac{\partial^3 U}{\partial x \partial y^2} + (B_{12}^s + 2B_{66}^s) \frac{\partial^3 V}{\partial x^2 \partial y} + B_{22}^s \frac{\partial^3 V}{\partial y^3} \\ & - \left[ D_{11}^s + \frac{1}{2} (A_m + B_m) \right] \frac{\partial^4 W_b}{\partial x^4} - (2D_{12}^s + 4D_{66}^s) \frac{\partial^4 W_b}{\partial x^2 \partial y^2} - \left[ D_{22}^s + \frac{1}{2} (A_m + B_m) \right] \frac{\partial^4 W_b}{\partial y^4} \\ & + \left( A_{55}^s + \frac{1}{4} H_m \right) \frac{\partial^2 W_s}{\partial x^2} + \left( A_{44}^s + \frac{1}{4} H_m \right) \frac{\partial^2 W_s}{\partial y^2} - \left[ H_{11} + \frac{1}{4} (A_m + 2B_m + C_m) \right] \frac{\partial^4 W_s}{\partial x^4} \\ & - (2H_{12} + 4H_{66}) \frac{\partial^4 W_s}{\partial x^2 \partial y^2} - \left[ H_{22} + \frac{1}{4} (A_m + 2B_m + C_m) \right] \frac{\partial^4 W_s}{\partial y^4} \\ & + \left( L_{13}^s + A_{55}^s - \frac{1}{4} H_m \right) \frac{\partial^2 \varphi_z}{\partial x^2} + \left( L_{23}^s + A_{44}^s - \frac{1}{4} H_m \right) \frac{\partial^2 \varphi_z}{\partial y^2} - \frac{1}{4} (D_m + E_m) \left( \frac{\partial^4 \varphi_z}{\partial x^4} + \frac{\partial^4 \varphi_z}{\partial y^4} \right) \\ & + P_0 + q - \frac{\partial^2 P_{xx}^T}{\partial x^2} - \frac{\partial^2 P_{yy}^T}{\partial y^2} = I_0 (\ddot{W}_b + \ddot{W}_s) + J_1 \left( \frac{\partial \ddot{U}}{\partial x} + \frac{\partial \ddot{V}}{\partial y} \right) - J_2 \nabla^2 \ddot{W}_b - K_2 \nabla^2 \ddot{W}_s + J_0 \ddot{\varphi}_z \end{aligned} \quad (24d)$$

$$\begin{aligned}
& -K_{13} \frac{\partial U}{\partial x} - K_{23} \frac{\partial V}{\partial y} + L_{13} \frac{\partial^2 W_b}{\partial x^2} + L_{23} \frac{\partial^2 W_b}{\partial y^2} - \frac{1}{2} E_m \frac{\partial^4 W_b}{\partial x^4} - \frac{1}{2} E_m \frac{\partial^4 W_b}{\partial y^4} \\
& + \left( L_{13}^s + A_{55}^s - \frac{1}{4} H_m \right) \frac{\partial^2 W_s}{\partial x^2} + \left( L_{23}^s + A_{44}^s - \frac{1}{4} H_m \right) \frac{\partial^2 W_s}{\partial y^2} - \frac{1}{4} (E_m + D_m) \frac{\partial^4 W_s}{\partial x^4} \\
& - \frac{1}{4} (E_m + D_m) \frac{\partial^4 W_s}{\partial y^4} - Z_{33} \varphi_z + \left( A_{55}^s + \frac{1}{4} H_m \right) \frac{\partial^2 \varphi_z}{\partial x^2} \\
& + \left( A_{44}^s + \frac{1}{4} H_m \right) \frac{\partial^2 \varphi_z}{\partial y^2} - \frac{1}{4} F_m \left( \frac{\partial^4 \varphi_z}{\partial x^4} + \frac{\partial^4 \varphi_z}{\partial y^4} \right) + gq + O_{zz}^T = J_0 (\ddot{W}_b + \ddot{W}_s) + K_0 \ddot{\varphi}_z
\end{aligned} \tag{24e}$$

### 3. Analytical solution

Based on the Navier approach, the displacements are expressed in terms of Fourier series as:

$$U(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{nm} \cos \alpha x \sin \beta y \tag{25a}$$

$$V(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} V_{nm} \sin \alpha x \cos \beta y \tag{25b}$$

$$W_b(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{bnm} \sin \alpha x \sin \beta y \tag{25c}$$

$$W_s(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{smn} \sin \alpha x \sin \beta y \tag{25d}$$

$$W_z(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{znm} \sin \alpha x \sin \beta y \tag{25e}$$

where  $\alpha = m\pi / a, \beta = n\pi / b$  and  $(U_{nm}, V_{nm}, W_{bnm}, W_{smn}, W_{znm})$  are coefficients. Similarly, mechanical/thermal loads are expanded as:

$$q(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} q_{nm} \sin \alpha x \sin \beta y \tag{26a}$$

$$\Delta T_i(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \Delta T_{imn} \sin \alpha x \sin \beta y, (i = 1, 2) \tag{26b}$$

and

$$(q_{nm}, \Delta T_{imn}) = (q_0, \Delta T_i) \text{ for sinusoidally distributed loads} \tag{27a}$$

$$(q_{nm}, \Delta T_{imn}) = \frac{16}{nm\pi^2} (q_0, \Delta T_i) \text{ for uniformly distributed loads} \tag{27b}$$

Substituting Eqs. (23) and (24) to Eq. (18), governing equations can be rewritten as:

$$(k_{ij} - m_{ij}\omega^2) \{u_{jmn}\} = \{F_{jmn}\}, \text{ where } k_{ij} = k_{ji}; m_{ij} = m_{ji} (i, j = \overline{1, 5}) \tag{28}$$

where

$$\begin{aligned}
k_{11} &= -A_{11}\alpha^2 - A_{66}\beta^2 - \frac{1}{4}A_m(\alpha^2\beta^2 + \beta^4); \quad k_{12} = -(A_{12} + A_{66})\alpha\beta + \frac{1}{4}A_m(\alpha^3\beta + \alpha\beta^3); \\
k_{13} &= B_{11}\alpha^3 + (B_{12} + 2B_{66})\alpha\beta^2; \quad k_{14} = B_{11}^s\alpha^3 + (B_{12}^s + 2B_{66}^s)\alpha\beta^2; \quad k_{15} = K_{13}\alpha \\
k_{22} &= -A_{22}\beta^2 - A_{66}\alpha^2 - \frac{1}{4}A_m(\alpha^4 + \alpha^2\beta^2); \quad k_{23} = B_{22}\beta^3 + (B_{12} + 2B_{66})\alpha^2\beta; \\
k_{24} &= B_{22}^s\beta^3 + (B_{12}^s + 2B_{66}^s)\alpha^2\beta; \quad k_{25} = K_{23}\beta \\
k_{33} &= -(D_{11} + A_m)\alpha^4 - (2D_{12} + 4D_{66} + 2A_m)\alpha^2\beta^2 - (D_{22} + A_m)\beta^4 - (\alpha^2 + \beta^2)P_0 \\
k_{34} &= -\left[D_{11}^s + \frac{1}{2}(A_m + B_m)\right]\alpha^4 - [2D_{12}^s + 4D_{66}^s + (A_m + B_m)]\alpha^2\beta^2 - \left[D_{22}^s + \frac{1}{2}(A_m + B_m)\right]\beta^4 \\
&\quad - (\alpha^2 + \beta^2)P_0 \\
k_{35} &= -L_{13}\alpha^2 - L_{23}\beta^2 - \frac{1}{2}E_m(\alpha^2 + \beta^2)^2 \\
k_{44} &= -\left(A_{55}^s - \frac{1}{4}H_m\right)\alpha^2 - \left(A_{44}^s + \frac{1}{4}H_m\right)\beta^2 - \left(H_{11} + \frac{1}{4}(A_m + 2B_m + C_m)\right)\alpha^4 - (2H_{12} + 4H_{66})\alpha^2\beta^2 \\
&\quad - \left(H_{22} + \frac{1}{4}(A_m + 2B_m + C_m)\right)\beta^4 - (\alpha^2 + \beta^2)P_0 \\
k_{45} &= -\left(L_{13}^s + A_{55}^s - \frac{1}{4}H_m\right)\alpha^2 - \left(L_{23}^s + A_{44}^s - \frac{1}{4}H_m\right)\beta^2 - \frac{1}{4}(D_m + E_m)(\alpha^2 + \beta^2)^2 \\
k_{55} &= -Z_{33} - \left(A_{55}^s + \frac{1}{4}H_m\right)\alpha^2 - \left(A_{44}^s + \frac{1}{4}H_m\right)\beta^2 - \frac{1}{4}F_m(\alpha^2 + \beta^2)^2 \tag{29a}
\end{aligned}$$

$$\begin{aligned}
m_{11} &= I_0; \quad m_{13} = -I_1\alpha; \quad m_{14} = -J_1\alpha; \quad m_{22} = I_0; \quad m_{23} = -I_1\beta; \quad m_{24} = -J_1\beta \\
m_{33} &= I_0 + I_2(\alpha^2 + \beta^2); \quad m_{34} = I_0 + J_2(\alpha^2 + \beta^2); \quad m_{35} = J_0 \\
m_{44} &= I_0 + K_2(\alpha^2 + \beta^2); \quad m_{45} = J_0; \quad m_{45} = K_0; \quad m_{12} = m_{15} = m_{25} = 0 \tag{29b}
\end{aligned}$$

$$\begin{aligned}
F_j &= \{F_1, F_2, F_3, F_4, F_5\} \\
F_1 &= -\alpha\left(A_t\Delta T_1 + B_t\frac{\Delta T_2}{h}\right); \quad F_2 = -\beta\left(A_t\Delta T_1 + B_t\frac{\Delta T_2}{h}\right); \quad F_3 = (\alpha^2 + \beta^2)\left(B_t\Delta T_1 + D_t\frac{\Delta T_2}{h}\right) \\
F_4 &= (\alpha^2 + \beta^2)\left(B_t^s\Delta T_1 + D_t^s\frac{\Delta T_2}{h}\right); \quad F_5 = \left(C_t\Delta T_1 + C_t^s\frac{\Delta T_2}{h}\right) \tag{29c}
\end{aligned}$$

In bending behaviour, the mid-plane displacements are simply calculated by  $\{u_{jmn}\} = k_{ij} \setminus \{F_{jmn}\}$ , where

$(m, n) = (1, 1)$  for the in-plane sinusoidal temperature and  $(m, n) = (\overline{1-19}, \overline{1-19})$  for the in-plane uniform

temperature. It should be noted that the increase of temperature results in the change of material properties and the thermal stress resultants, which leads to the forced vibration and nonlinear buckling analysis. In this paper, only in-plane thermal resultant (axial force) is considered which enable to solve the problems using eigenvalue algorithm. The thermal vibration and buckling behaviours are similar to mechanical behaviours including the temperature-dependent material properties, thus a trial and error procedure needs to be applied to obtain the critical buckling temperatures.

#### 4. Numerical results and discussion

In this session, numerical examples are carried out to study the bending, vibration and buckling behaviours of FG sandwich microplates under mechanical and thermal loads. Firstly, verification is presented for microplates under mechanical loads and macroplates under thermal loads. Differences between constitutive relations as well as between Third-order Shear Deformation Theory (TSDT) and Sinusoidal Shear Deformation Theory (SSDT) is also discussed. Parameter study is then conducted to investigate the thermal behaviours of FG sandwich microplates. In these examples, effects of geometric configurations and temperatures on microplates are investigated using both 2D and quasi-3D models. Apart from the temperature-independent (TID) form, which is applied to verify the present solution, the temperature-dependent (TD) form is considered in the rest of this work. Material properties for mechanical and TD coefficients for thermal analysis are presented in Tables 1 and 2. Non-dimensional expressions used in this paper are:

Bending:

Mechanical load:

$$\text{Deflection: } \bar{w}(z) = \frac{10E_c h^3}{q_0 a^4} u_3 \left( \frac{a}{2}, \frac{b}{2}, z \right) \quad (30a)$$

$$\text{Stress: } \bar{\sigma}_{xx} = \frac{h\sigma_{xx}}{aq_0}, \quad \bar{\sigma}_{xz} = \frac{h\sigma_{xz}}{aq_0} \quad (30b)$$

Thermal load:

$$\text{Deflection: } \bar{w}(z) = \frac{h}{\alpha_0 \Delta T_2 a^2} u_3 \left( \frac{a}{2}, \frac{b}{2}, z \right) \quad (30c)$$

$$\text{Stress: } \bar{\sigma}_{xx} = \frac{h^2}{\alpha_0 \Delta T_2 E_0 a^2} \sigma_{xx} \left( \frac{a}{2}, \frac{b}{2}, z \right) \quad (30d)$$

where  $E_0 = 1\text{GPa}$ ,  $\alpha_0 = 10^{-6} \text{ } ^\circ\text{C}^{-1}$

$$\text{Vibration: } \hat{\omega} = \frac{\omega a^2}{h} \sqrt{\frac{\rho_c}{E_c}}, \quad \bar{\omega} = \frac{\omega a^2}{h} \sqrt{\frac{\rho_0 (1 - \nu_0^2)}{E_0}} \quad (30e)$$

$$\text{Buckling:} \quad \bar{P}_{cr} = P_{cr} \frac{a^2}{E_0 h^3} \quad (30f)$$

where  $E_0$ ,  $\rho_0$  and  $\nu_0$  are the Young's modulus, mass density and Poisson's ratio of metal at 300K.

#### 4.1. Verification:

##### a. Static bending analysis:

Table 3 validates the obtained solutions of transverse displacement  $\bar{w}$  and normal stress  $\bar{\sigma}_{xx}$  under mechanical load for microplates. The sinusoidal mechanical load is applied to Al/Al<sub>2</sub>O<sub>3</sub> plate with different slenderness ratios and power-law indices. Under mechanical load, the present quasi-3D results are in good agreement with other quasi-3D ones. Considering 2D models, Lei et al. [38] degenerated quasi-3D to 2D model by setting  $W_z$  to zero, which results in a slight difference in stress  $\bar{\sigma}_{xx}$  compared to present solutions and those obtained by Thai and Vo [25]. It is also seen that quasi-3D models provide higher deflections for small-scale plates but lower values for thicker plates compared to 2D models. Further verification is presented for thermal bending of FG sandwich macroplates. By applying the linear temperature through the thickness, the deflections and stresses of Ti-6Al-4V/ZrO<sub>2</sub> plates are presented in Table 4. It is seen that the present TSDT and SSDT results are in excellent agreement with those obtained from Tounsi et al. [39] while the quasi-3D ones agree well with those reported by Mantari and Granados [40]. It should be noted that the 2D constitutive relation, i.e.  $[Q_{ii}^{2D}, Q_{ij}^{2D}]$ , was applied to the quasi-3D solution in [40]. In this example, applying quasi-3D theories not only increases the stiffness but also induces the load component  $F_5$  which is caused by z-direction thermal stretching. Therefore, thermal deflections achieved from quasi-3D models together with  $[Q_{ii}^{3D}, Q_{ij}^{3D}]$  are higher than those from 2D models. At the same time, between SSDT and TSDT which are employed to quasi-3D models, the former provides higher stress magnitudes and the difference is much more pronounced in thermal case. Those values from SSDT quasi-3D model are comparable to the results obtained from 2D solutions. In this paper, SSDT is applied to static bending behaviours while TSDT is applied to vibration and buckling analysis.

##### b. Vibration and buckling analysis:

Fundamental frequencies of Al/Al<sub>2</sub>O<sub>3</sub> microplates are compared with those of Lei et al. [38] in Table 5 and presented in Fig. 2 for various slenderness and material length scale ratios. Excellent agreement can be observed. In addition, the natural frequencies reduce with higher material length scale ratios and level out as  $h/l \geq 20$ . With respect to the thermal effect, the vibration of Si<sub>3</sub>N<sub>4</sub>/SUS304 macroplates ( $a/h=10$ ) under uniform temperature is verified with those given in [41] considering TID and TD forms in Table 6. In this example, the equivalent axial load caused by the thermal stress is calculated



with/without the transverse thermal stress  $\sigma_{zz}^T$  (either  $[Q_{ii}^{3D}, Q_{ij}^{3D}]$  or  $[Q_{ii}^{2D}, Q_{ij}^{2D}]$  is utilised for thermal stress, respectively). The present TSDT results are almost the same with those in [41] while the quasi-3D solution provides lower natural frequencies, especially when  $\sigma_{zz}^T$  is included. The TD solution, which describes better the working status of materials, also displays lower natural frequencies compared to TID one.

Further verification is carried out for mechanical and thermal buckling behaviours of FG plates in Tables 7 and 8. As can be seen in Table 7, the present results of mechanical buckling agree well with those obtained from refined plate theories (RPT) [26, 28] and quasi-3D theory [28]. It is also shown that the inclusion of normal stretching effect leads to the higher critical buckling loads for macroplates but less significant to microplates. Regarding the thermal buckling behaviour, by applying the linear temperature which is expressed by  $T(x, y, z) = 305 + (1/2 + z/h)\Delta T$  [31], a good agreement can be seen for Al/Al<sub>2</sub>O<sub>3</sub> macroplates and thin microplates in Table 8. This temperature pattern, which assumes the temperature elevation occurs only at the top surface, can lead to a very high difference in the temperature at the bottom and top surfaces; therefore, the temperature distribution described in Eq. (4) and the exclusion of  $\sigma_{zz}^T$  are considered in the thermal vibration and buckling analysis.

#### 4.2. Parameter study:

##### a. Static bending analysis:

The deflection and stress of Ti-6Al-4V/ZrO<sub>2</sub> plates under various thickness scales and temperatures is presented in Fig. 3. In this figure, TID is considered to verify with those obtained from [40] for the macroplates under the higher temperatures. The small-scale effect can be seen for deflection and stress in microplates and is negligible for thicker plates ( $h \geq 20l$ ) where a good agreement with those results for macroplates is observed. Regarding the thickness stretching effect, the results from the quasi-3D and 2D models agree well at ambient temperature but the significant difference between them is observed at elevated temperature. In addition, under lower temperature, the positive deflection can be seen while the negative deflection is obtained with increasing temperature. This can be explained by the difference in thermal expansions of Ti-6Al-4V and ZrO<sub>2</sub>. At the lower temperature, the linear temperature causes an expansion on the upper face (ceramic – ZrO<sub>2</sub>) whereas the metal has not been heated up. This results in a coupled force pushing the plate upward. At the higher temperature  $\Delta T_1$ , due to the higher thermal expansion coefficient, the metal surface stretches more than the ceramic one which creates a reverse moment pushing the plate downward. This change is illustrated by the transverse displacements obtained from quasi-3D and 2D models for various thickness scales in Fig. 4. It is seen that the difference between these solutions are more profound under thermal loads than under mechanical loads.

Tables 9 and 10 present the non-dimensional deflection of  $\text{Si}_3\text{N}_4/\text{SUS304}$  sandwich microplates under uniform and linear temperatures through the thickness. No displacement is shown in Table 9 for homogeneous plates (FG or homogeneous-core sandwich plate with  $p = 0$ ) due to the symmetry of material and thermal stress under uniform temperature. The deflection increases with respect to temperature but in different ways, upward for the ceramic-core plates and downward for the others. Also, less deflection can be observed for smaller scale plates under thermal environment, which is similar to the case of mechanical loads. At the macro scale, smallest displacement is observed in metal-core plates, especially under uniform temperature. At the micro scale, the homogeneous-core plates induce lower deflection under uniform temperature, but higher deflection under linear temperature compared to the FG/ FG-core plates. To compare the quasi-3D and 2D models in evaluating thermal stresses, Fig. 5 displays the through-the-thickness normal and shear stresses of sandwich microplates under thermal environment. The inclusion of stretching effect leads to the higher stresses and this difference is more significant at the top and bottom surfaces for the normal stress and at the middle surface for the shear stress. The stress distributions under uniform and linear temperatures are revealed in Fig. 6. The linear temperature results in the higher normal stress at the top surface but the lower at the bottom surface compared to the uniform temperature. In addition, the higher shear stresses are obtained under uniform temperature for FG-core plates but under linear temperature for the homogeneous-core plates. In increasing the material length scales ratio, the shear stresses reduce significantly and the normal stress also changes the distribution, from higher at the top to higher at the bottom.

*b. Vibration and buckling analysis:*

The natural frequencies under uniform and linear temperatures are presented for  $\text{Si}_3\text{N}_4/\text{SUS304}$  microplates ( $a/h = 10, p = 1$ ) in Fig. 7 with TD solution. In this example,  $[Q_{ii}^{3D}, Q_{ij}^{3D}]$  relation is applied for thermal stress in evaluating equivalent axial loads. As expected, the natural frequency decreases as increasing temperature up to zero at the critical buckling temperature. In addition, the difference between the natural frequencies under uniform and linear temperatures is minor, but the difference between 2D and quasi-3D solutions increases with elevated temperature. Considering the small-scale effect, the smaller material length scale ratios are considered, the higher natural frequencies are obtained. This can be explained that including the deviatoric part of the symmetric couple stress tensors in strain energy strengthen the stiffness of the microplates. The natural frequencies of sandwich microplates under thermal environment are presented in Fig. 8 and Tables 11-13 for FG-, ceramic- and metal- core, respectively. Under both uniform and linear temperatures, the highest natural frequencies can be seen in ceramic-core and the lowest in metal-core plates. In addition, the increase of power-law index leads to the lower natural frequencies in FG-core and ceramic-core plates but the higher natural frequencies in

metal-core plates. Finally, the critical buckling temperatures are tabulated in Table 14 for  $\text{Si}_3\text{N}_4/\text{SUS304}$  microplates. As expected, the higher values are observed for smaller material length scale ratios. The relation between the power-law index and the critical buckling temperature for these sandwich plates is similar to what is observed in vibration behaviours.

## 5. Conclusions

In this study, general quasi-3D and higher-order shear deformation models are developed to study the mechanical and thermal behaviours of FG sandwich microplates. Governing equations are derived from the variational principle based on the framework of the modified couple stress theory. The Navier solutions are applied to examine the bending, vibration and buckling behaviours of microplates under mechanical and thermal loads. These solutions reveal that the inclusion of small-scale effect increases the microplates' stiffness, especially for those with the thickness  $h < 20l$ . In addition, comprising thickness stretching strain in mechanical analysis results in higher stiffness for FG plates which leads to smaller deflection and higher natural frequencies as well as critical buckling loads, compared with the higher-order 2D models. Moreover, thickness stretching thermal strain also induces the out-of-plane thermal load in the thermal analysis which leads to the higher deflections, stresses as well as lower natural frequency and critical buckling temperature.

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## APPENDIX

$$N_{xx} = A_{11} \frac{\partial U}{\partial x} + A_{12} \frac{\partial V}{\partial y} - B_{11} \frac{\partial^2 W_b}{\partial x^2} - B_{12} \frac{\partial^2 W_b}{\partial y^2} - B_{11}^s \frac{\partial^2 W_s}{\partial x^2} - B_{12}^s \frac{\partial^2 W_s}{\partial y^2} + K_{13} W_z - \left( A_t \Delta T_1 + B_t \frac{\Delta T_2}{h} \right) \quad (A1)$$

$$N_{yy} = A_{12} \frac{\partial U}{\partial x} + A_{22} \frac{\partial V}{\partial y} - B_{12} \frac{\partial^2 W_b}{\partial x^2} - B_{22} \frac{\partial^2 W_b}{\partial y^2} - B_{12}^s \frac{\partial^2 W_s}{\partial x^2} - B_{22}^s \frac{\partial^2 W_s}{\partial y^2} + K_{23} W_z - \left( A_t \Delta T_1 + B_t \frac{\Delta T_2}{h} \right) \quad (A2)$$

$$N_{xy} = A_{66} \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) - 2B_{66} \frac{\partial^2 W_b}{\partial x \partial y} - 2B_{66}^s \frac{\partial^2 W_s}{\partial x \partial y} \quad (A3)$$

$$M_{xx} = B_{11} \frac{\partial U}{\partial x} + B_{12} \frac{\partial V}{\partial y} - D_{11} \frac{\partial^2 W_b}{\partial x^2} - D_{12} \frac{\partial^2 W_b}{\partial y^2} - D_{11}^s \frac{\partial^2 W_s}{\partial x^2} - D_{12}^s \frac{\partial^2 W_s}{\partial y^2} + L_{13} W_z - \left( B_t \Delta T_1 + D_t \frac{\Delta T_2}{h} \right) \quad (A4)$$

$$M_{yy} = B_{12} \frac{\partial U}{\partial x} + B_{22} \frac{\partial V}{\partial y} - D_{12} \frac{\partial^2 W_b}{\partial x^2} - D_{22} \frac{\partial^2 W_b}{\partial y^2} - D_{12}^s \frac{\partial^2 W_s}{\partial x^2} - D_{22}^s \frac{\partial^2 W_s}{\partial y^2} + L_{23} W_z - \left( B_t \Delta T_1 + D_t \frac{\Delta T_2}{h} \right) \quad (A5)$$

$$M_{xy} = B_{66} \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) - 2D_{66} \frac{\partial^2 W_b}{\partial x \partial y} - 2D_{66}^s \frac{\partial^2 W_s}{\partial x \partial y} \quad (A6)$$

$$P_{xx} = B_{11}^s \frac{\partial U}{\partial x} + B_{12}^s \frac{\partial V}{\partial y} - D_{11}^s \frac{\partial^2 W_b}{\partial x^2} - D_{12}^s \frac{\partial^2 W_b}{\partial y^2} - H_{11} \frac{\partial^2 W_s}{\partial x^2} - H_{12} \frac{\partial^2 W_s}{\partial y^2} + L_{13}^s W_z - \left( B_t^s \Delta T_1 + D_t^s \frac{\Delta T_2}{h} \right) \quad (A7)$$

$$P_{yy} = B_{12}^s \frac{\partial U}{\partial x} + B_{22}^s \frac{\partial V}{\partial y} - D_{12}^s \frac{\partial^2 W_b}{\partial x^2} - D_{22}^s \frac{\partial^2 W_b}{\partial y^2} - H_{12} \frac{\partial^2 W_s}{\partial x^2} - H_{22} \frac{\partial^2 W_s}{\partial y^2} + L_{23}^s W_z - \left( B_t^s \Delta T_1 + D_t^s \frac{\Delta T_2}{h} \right) \quad (A8)$$

$$P_{xy} = B_{66}^s \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) - 2D_{66}^s \frac{\partial^2 W_b}{\partial x \partial y} - 2H_{66} \frac{\partial^2 W_s}{\partial x \partial y} \quad (A9)$$

$$O_{zz} = K_{13} \frac{\partial U}{\partial x} + K_{23} \frac{\partial V}{\partial y} - L_{13} \frac{\partial^2 W_b}{\partial x^2} - L_{23} \frac{\partial^2 W_b}{\partial y^2} - L_{13}^s \frac{\partial^2 W_s}{\partial x^2} - L_{23}^s \frac{\partial^2 W_s}{\partial y^2} + Z_{33} W_z - \left( C_t \Delta T_1 + C_t^s \frac{\Delta T_2}{h} \right) \quad (A10)$$

$$Q_{xz} = A_{55}^s \left( \frac{\partial W_s}{\partial x} + \frac{\partial W_z}{\partial x} \right) \quad (A11)$$

$$Q_{yz} = A_{44}^s \left( \frac{\partial W_s}{\partial y} + \frac{\partial W_z}{\partial y} \right) \quad (A12)$$

$$R_{xx} = 2A_m \frac{\partial^2 W_b}{\partial x \partial y} + (A_m + B_m) \frac{\partial^2 W_s}{\partial x \partial y} + E_m \frac{\partial^2 W_z}{\partial x \partial y} \quad (A13)$$

$$R_{yy} = -2A_m \frac{\partial^2 W_b}{\partial x \partial y} - (A_m + B_m) \frac{\partial^2 W_s}{\partial x \partial y} - E_m \frac{\partial^2 W_z}{\partial x \partial y} \quad (A14)$$

$$R_{xy} = A_m \frac{\partial^2 W_b}{\partial y^2} + \frac{1}{2} (A_m + B_m) \frac{\partial^2 W_s}{\partial y^2} + \frac{1}{2} E_m \frac{\partial^2 W_z}{\partial y^2} - A_m \frac{\partial^2 W_b}{\partial x^2} - \frac{1}{2} (A_m + B_m) \frac{\partial^2 W_s}{\partial x^2} - \frac{1}{2} E_m \frac{\partial^2 W_z}{\partial x^2} \quad (A15)$$

$$R_{xz} = \frac{1}{2} A_m \frac{\partial^2 V}{\partial x^2} - \frac{1}{2} A_m \frac{\partial^2 U}{\partial x \partial y} - \frac{1}{2} G_m \frac{\partial W_s}{\partial y} + \frac{1}{2} G_m \frac{\partial W_z}{\partial y} \quad (A16)$$

$$R_{yz} = \frac{1}{2} A_m \frac{\partial^2 V}{\partial x \partial y} - \frac{1}{2} A_m \frac{\partial^2 U}{\partial y^2} + \frac{1}{2} G_m \frac{\partial W_s}{\partial x} - \frac{1}{2} G_m \frac{\partial W_z}{\partial x} \quad (A17)$$

$$S_{xx} = 2B_m \frac{\partial^2 W_b}{\partial x \partial y} + (B_m + C_m) \frac{\partial^2 W_s}{\partial x \partial y} + D_m \frac{\partial^2 W_z}{\partial x \partial y} \quad (A18)$$

$$S_{yy} = -2B_m \frac{\partial^2 W_b}{\partial x \partial y} - (B_m + C_m) \frac{\partial^2 W_s}{\partial x \partial y} - D_m \frac{\partial^2 W_z}{\partial x \partial y} \quad (A19)$$

$$S_{xy} = B_m \frac{\partial^2 W_b}{\partial y^2} + \frac{1}{2} (B_m + C_m) \frac{\partial^2 W_s}{\partial y^2} + \frac{1}{2} D_m \frac{\partial^2 W_z}{\partial y^2} - B_m \frac{\partial^2 W_b}{\partial x^2} - \frac{1}{2} (B_m + C_m) \frac{\partial^2 W_s}{\partial x^2} - \frac{1}{2} D_m \frac{\partial^2 W_z}{\partial x^2} \quad (A20)$$

$$T_{xx} = 2E_m \frac{\partial^2 W_b}{\partial x \partial y} + (E_m + D_m) \frac{\partial^2 W_s}{\partial x \partial y} + F_m \frac{\partial^2 W_z}{\partial x \partial y} \quad (A21)$$

$$T_{yy} = -2E_m \frac{\partial^2 W_b}{\partial x \partial y} - (E_m + D_m) \frac{\partial^2 W_s}{\partial x \partial y} - F_m \frac{\partial^2 W_z}{\partial x \partial y} \quad (A22)$$

$$T_{xy} = E_m \frac{\partial^2 W_b}{\partial y^2} + \frac{1}{2} (E_m + D_m) \frac{\partial^2 W_s}{\partial y^2} + \frac{1}{2} F_m \frac{\partial^2 W_z}{\partial y^2} - E_m \frac{\partial^2 W_b}{\partial x^2} - \frac{1}{2} (E_m + D_m) \frac{\partial^2 W_s}{\partial x^2} - \frac{1}{2} F_m \frac{\partial^2 W_z}{\partial x^2} \quad (A23)$$

$$X_{xz} = -\frac{1}{2} G_m \frac{\partial^2 V}{\partial x^2} + \frac{1}{2} G_m \frac{\partial^2 U}{\partial x \partial y} + \frac{1}{2} H_m \frac{\partial W_s}{\partial y} - \frac{1}{2} H_m \frac{\partial W_z}{\partial y} \quad (A24)$$

$$X_{yz} = -\frac{1}{2} G_m \frac{\partial^2 V}{\partial x \partial y} + \frac{1}{2} G_m \frac{\partial^2 U}{\partial y^2} - \frac{1}{2} H_m \frac{\partial W_s}{\partial x} + \frac{1}{2} H_m \frac{\partial W_z}{\partial x} \quad (A25)$$

where

$$(A_{ij}, A_{ij}^s, B_{ij}, B_{ij}^s, D_{ij}, D_{ij}^s, H_{ij}) = \int_{-h/2}^{h/2} [1, g^2(z), z, f(z), z^2, f(z)z, f^2(z)] Q_{ij} dz \quad (A26)$$

$$(K_{ij}, L_{ij}, L_{ij}^s, Z_{ij}) = \int_{-h/2}^{h/2} \left[ 1, z, f(z), \frac{\partial g(z)}{\partial z} \right] \frac{\partial g(z)}{\partial z} Q_{ij} dz \quad (A27)$$

$$(A_m, B_m, C_m, D_m, E_m, F_m, G_m, H_m) = \int_{-h/2}^{h/2} l^2 \mu \left( 1, \frac{\partial f(z)}{\partial z}, \left[ \frac{\partial f(z)}{\partial z} \right]^2, \frac{\partial f(z)}{\partial z} g(z), g(z), [g(z)]^2, \frac{\partial g(z)}{\partial z}, \left[ \frac{\partial g(z)}{\partial z} \right]^2 \right) dz \quad (A28)$$

$$A_i = \int_{-h/2}^{h/2} \alpha(z, T) (Q_{11} + 2Q_{12}) dz ; B_i = \int_{-h/2}^{h/2} \alpha(z, T) (Q_{11} + 2Q_{12}) z dz ; \quad (A29)$$

$$B_t^s = \int_{-h/2}^{h/2} \alpha(z, T)(Q_{11} + 2Q_{12})f(z)dz; C_t = \int_{-h/2}^{h/2} \alpha(z, T)(Q_{11} + 2Q_{12})\frac{d^2 f}{dz^2} dz; \quad (\text{A30})$$

$$C_t^s = \int_{-h/2}^{h/2} \alpha(z, T)(Q_{11} + 2Q_{12})\frac{d^2 f}{dz^2} z dz; D_t = \int_{-h/2}^{h/2} \alpha(z, T)(Q_{11} + 2Q_{12})z^2 dz; \quad (\text{A31})$$

$$D_t^s = \int_{-h/2}^{h/2} \alpha(z, T)(Q_{11} + 2Q_{12})f(z)z dz \quad (\text{A32})$$

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Table 14: Critical buckling temperatures  $\Delta T_{cr} [K]$  of (1-2-2)  $\text{Si}_3\text{N}_4/\text{SUS304}$  microplates  $(a/h = 20)$ .

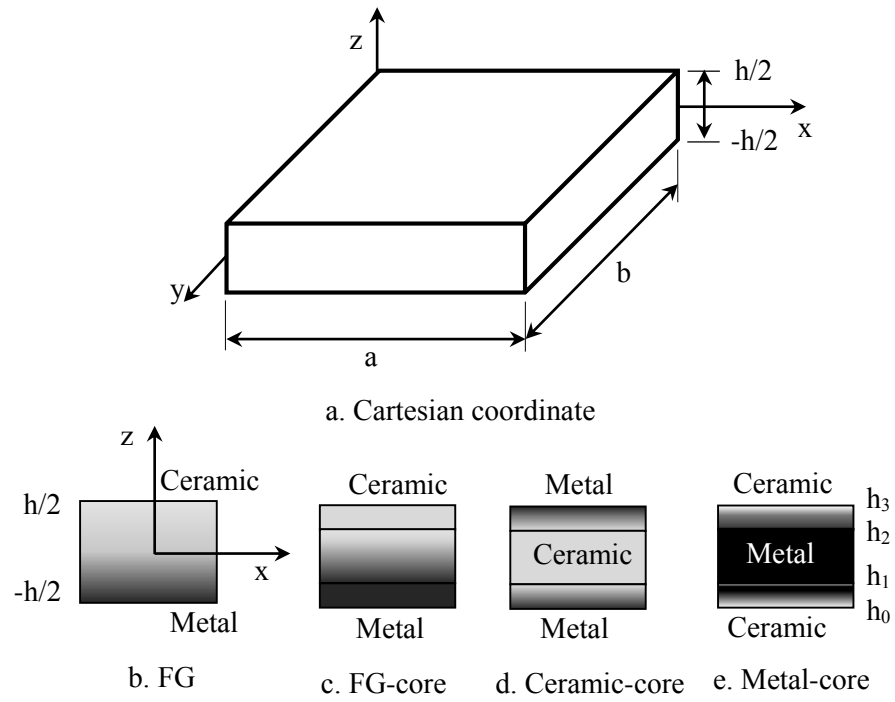


Fig. 1: Coordinate and cross-section of FG/FG sandwich plates.

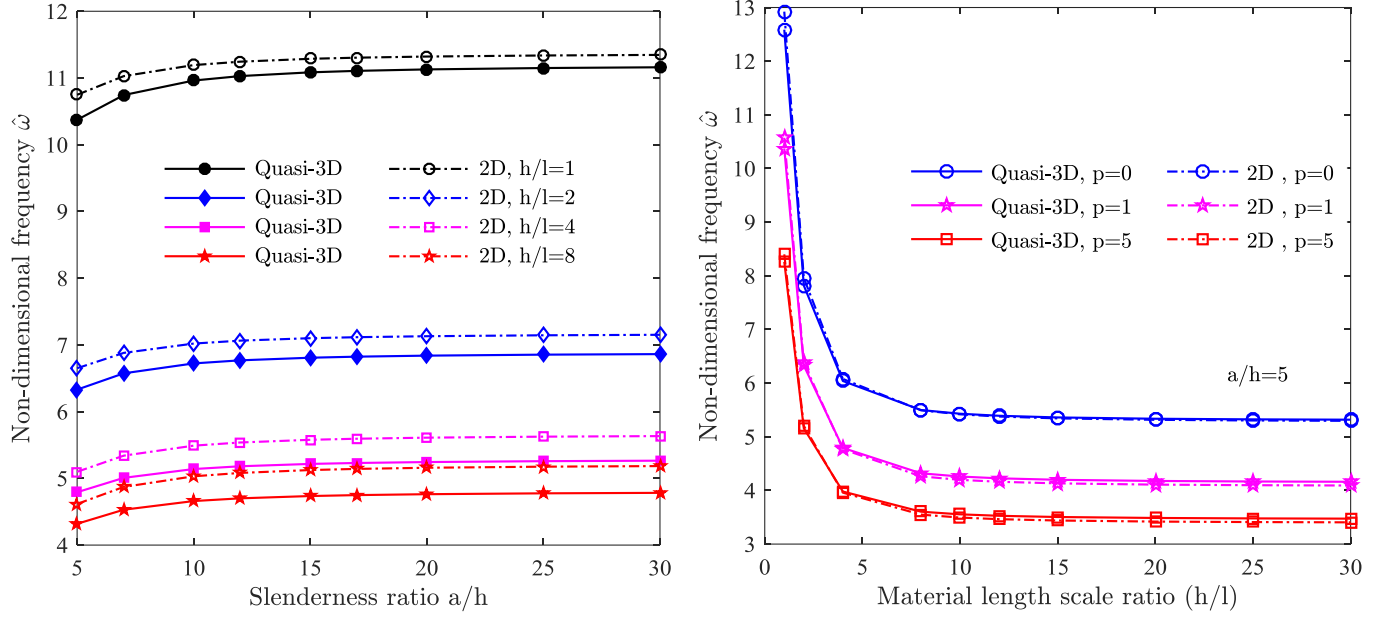
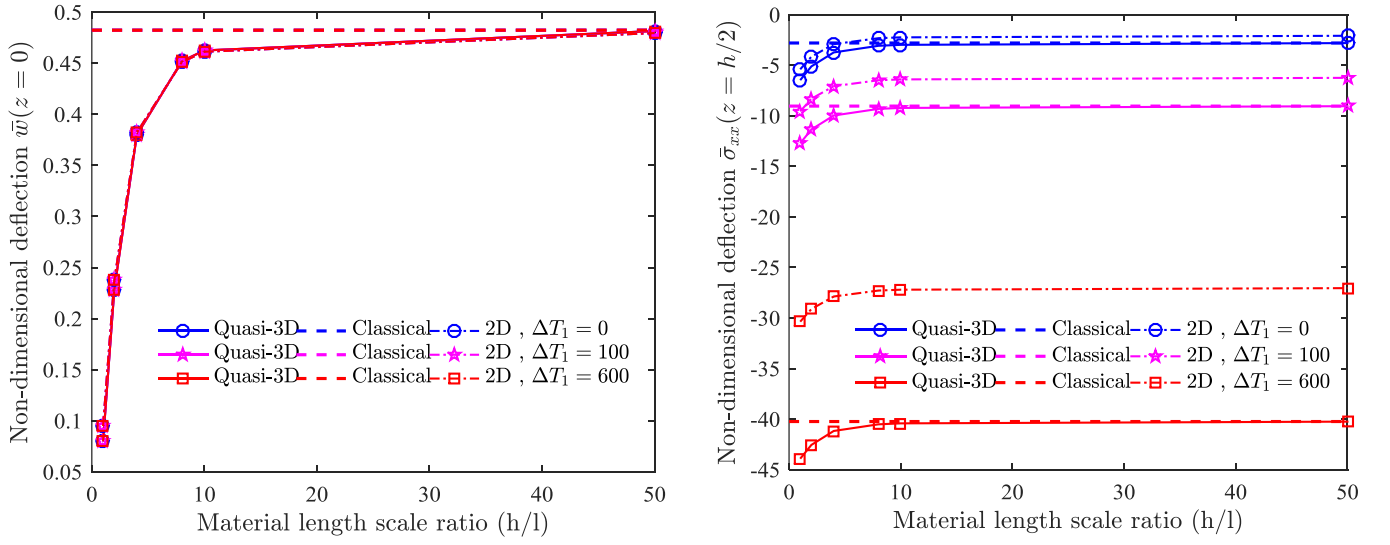
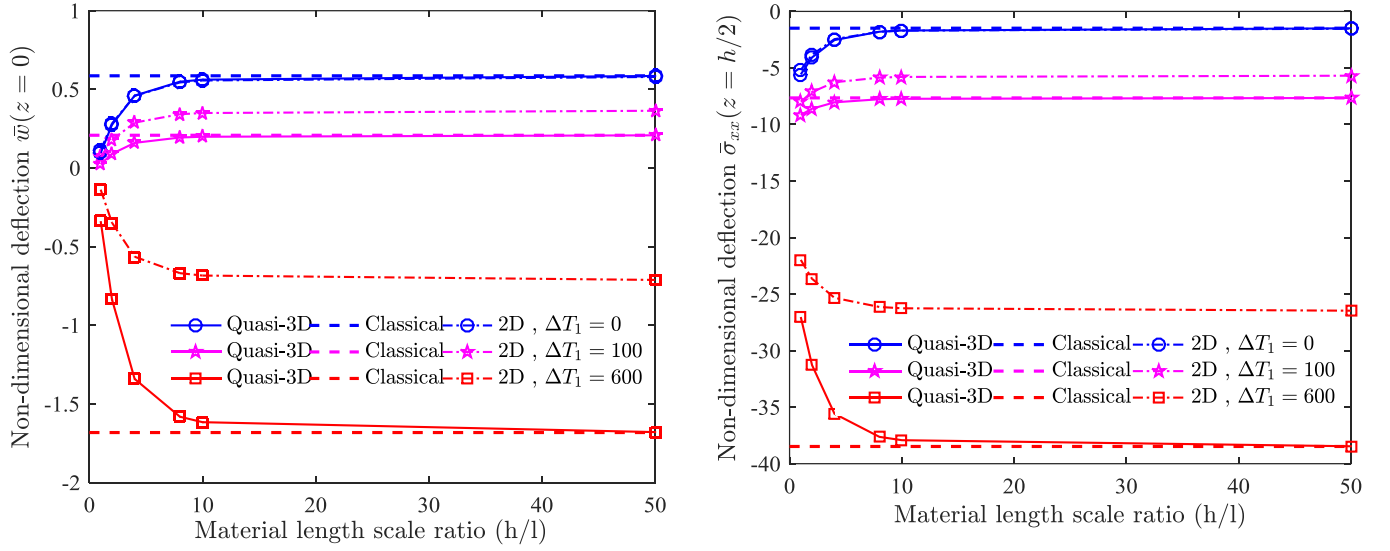


Fig. 2: Non-dimensional frequencies of Al/Al<sub>2</sub>O<sub>3</sub> microplates with various slenderness and material length scale ratios.



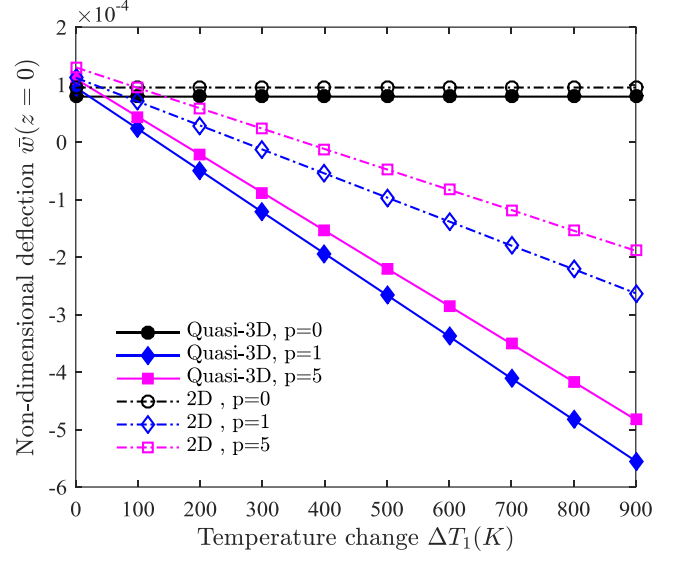
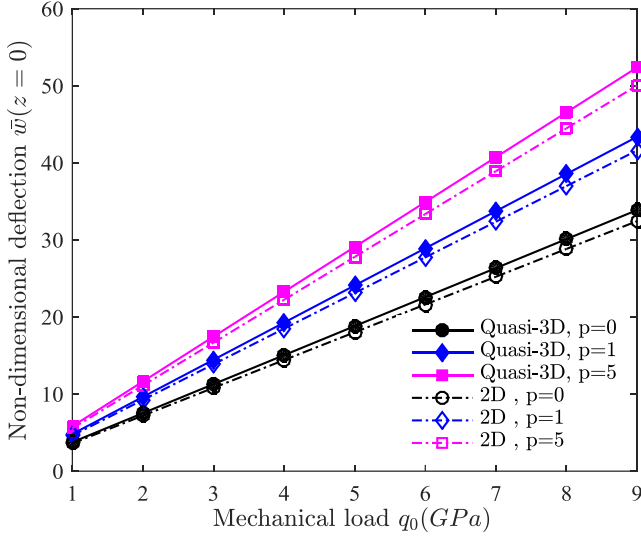
a)  $p = 0$



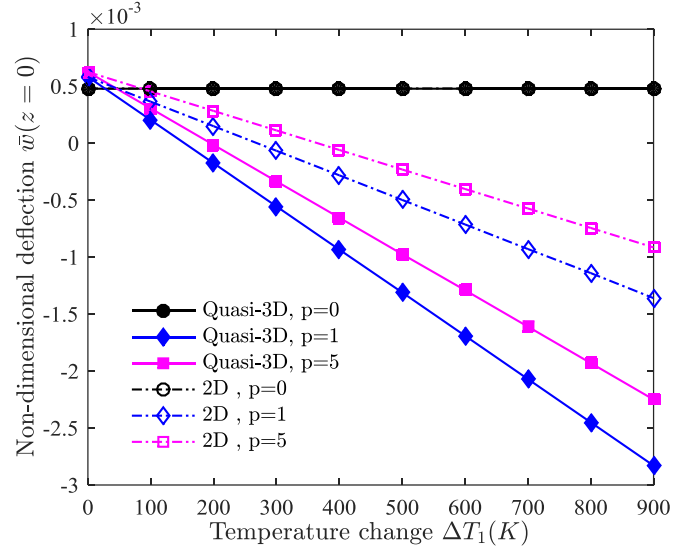
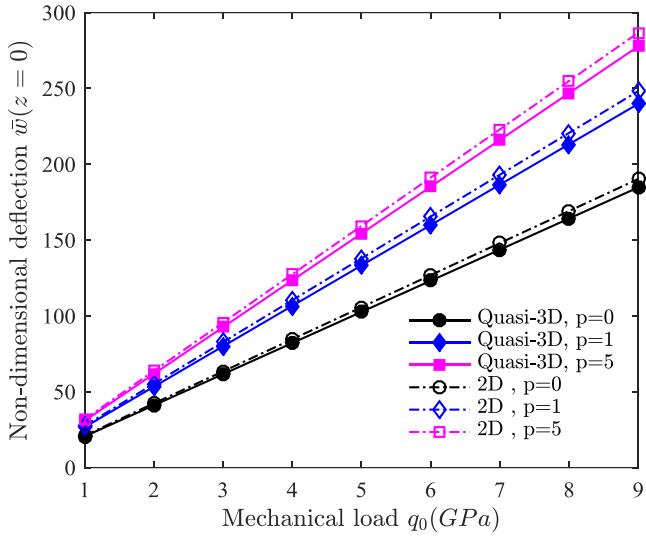
b)  $p = 1$

Fig. 3: Non-dimensional deflections and stresses of Ti-6Al-4V/ZrO<sub>2</sub> microplates for various material length scale ratios and temperatures.



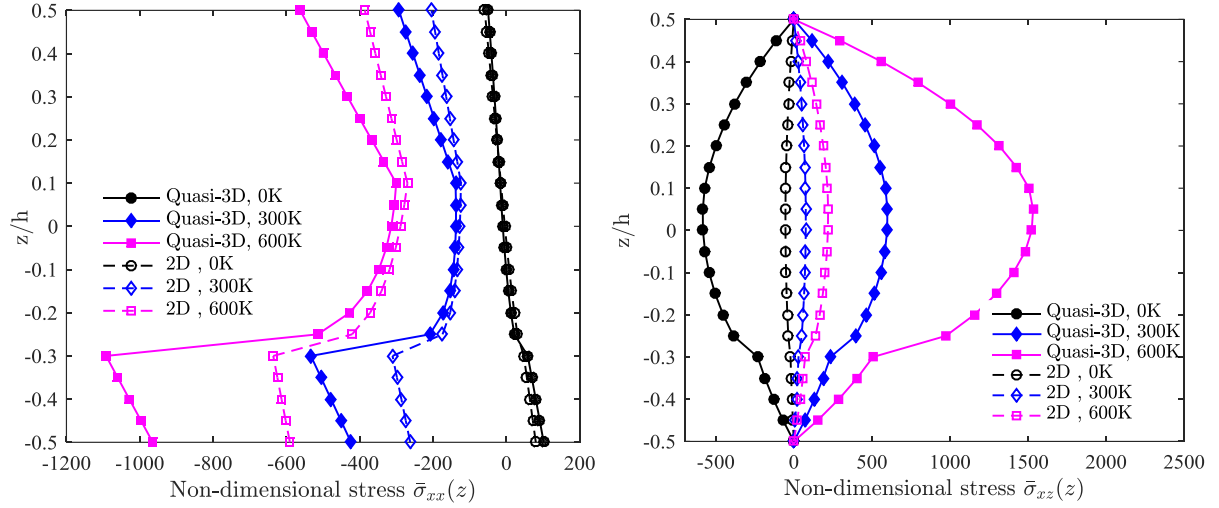


a)  $h/l=1$

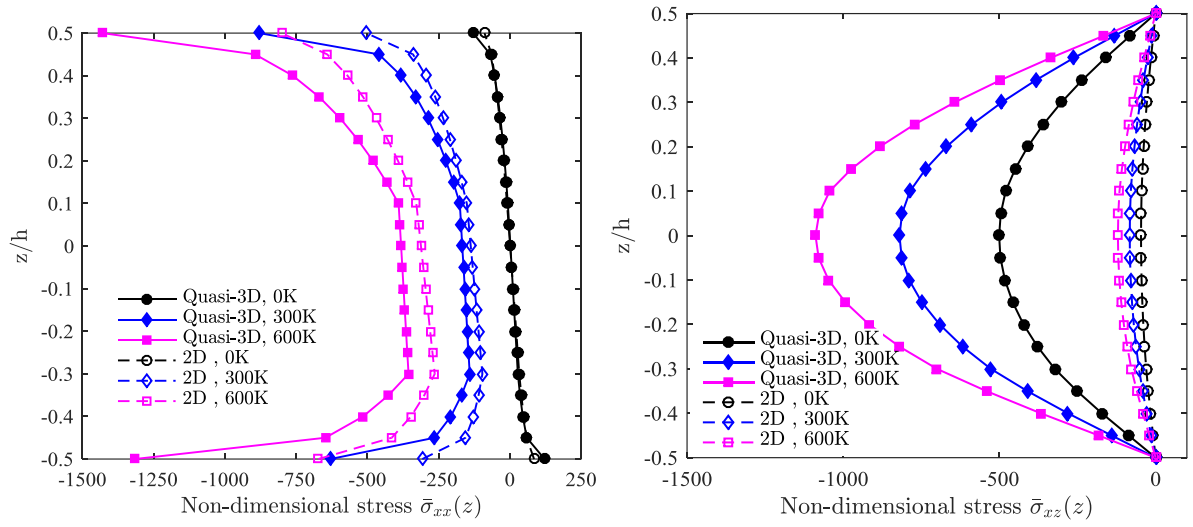


b) Classical

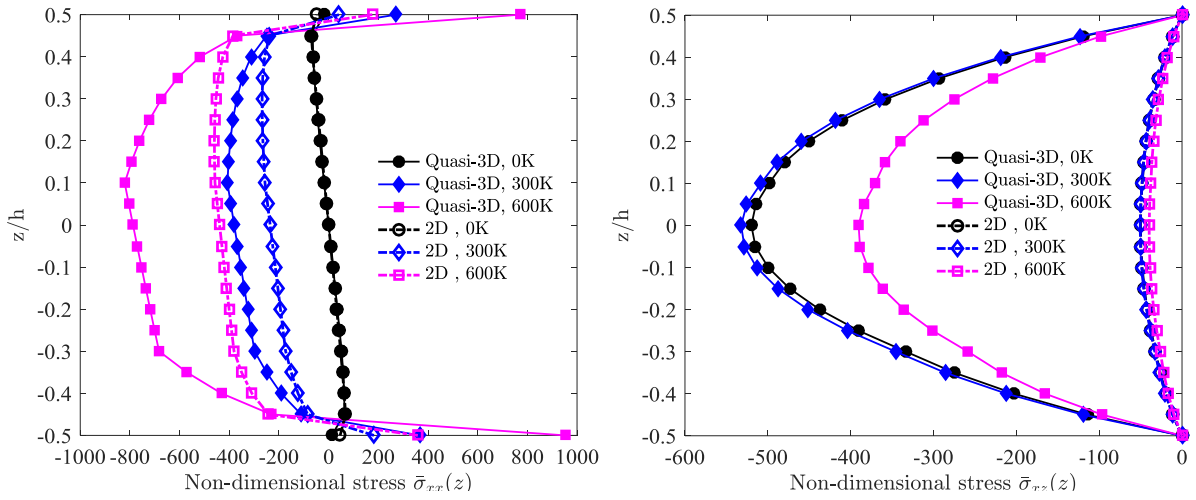
Fig. 4: Non-dimensional deflections of Ti-6Al-4V/ZrO2 microplates under various mechanical and thermal loads.



a. FG-core sandwich plates

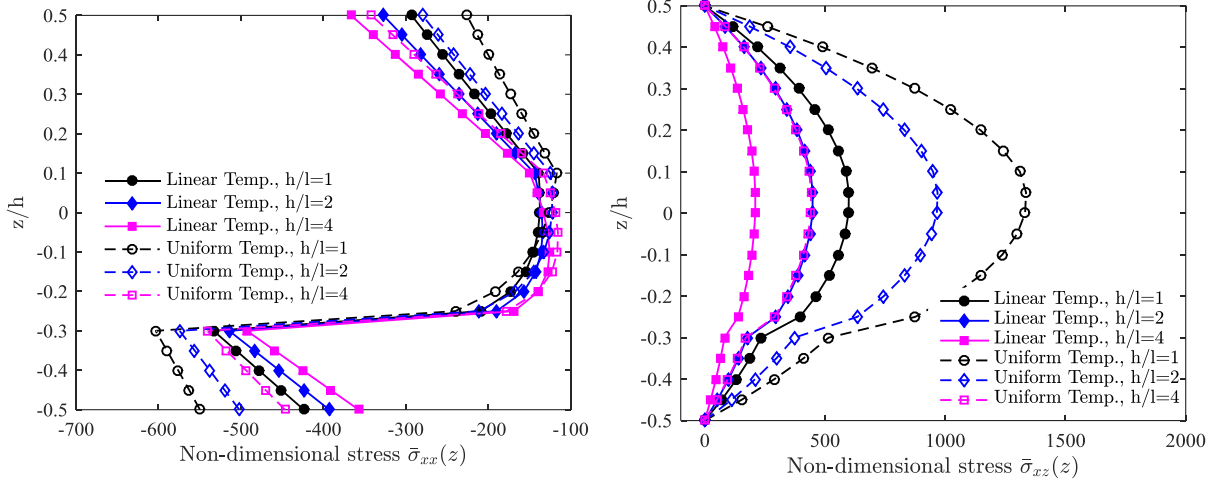


b. Ceramic-core sandwich plates

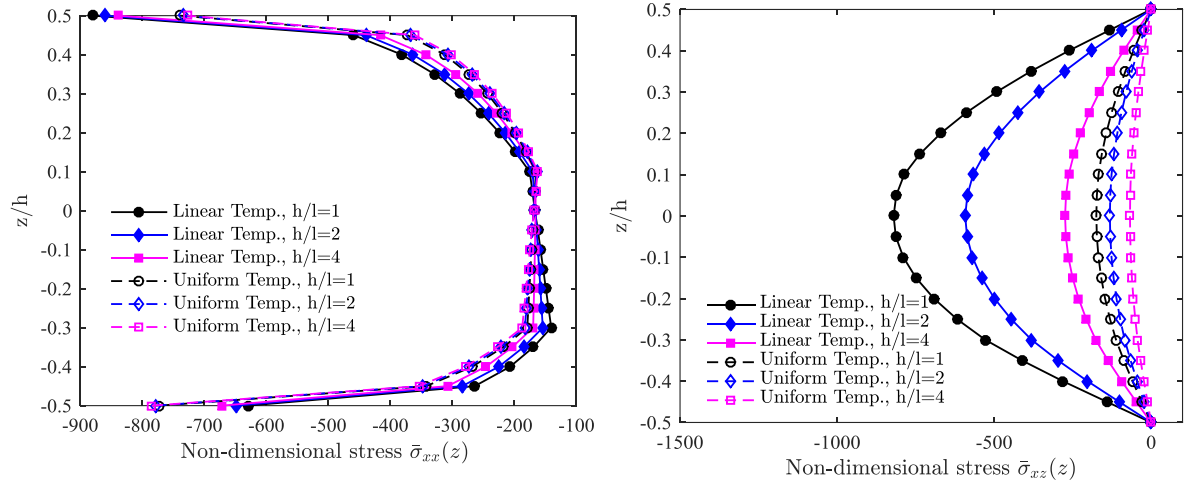


c. Metal-core sandwich plates

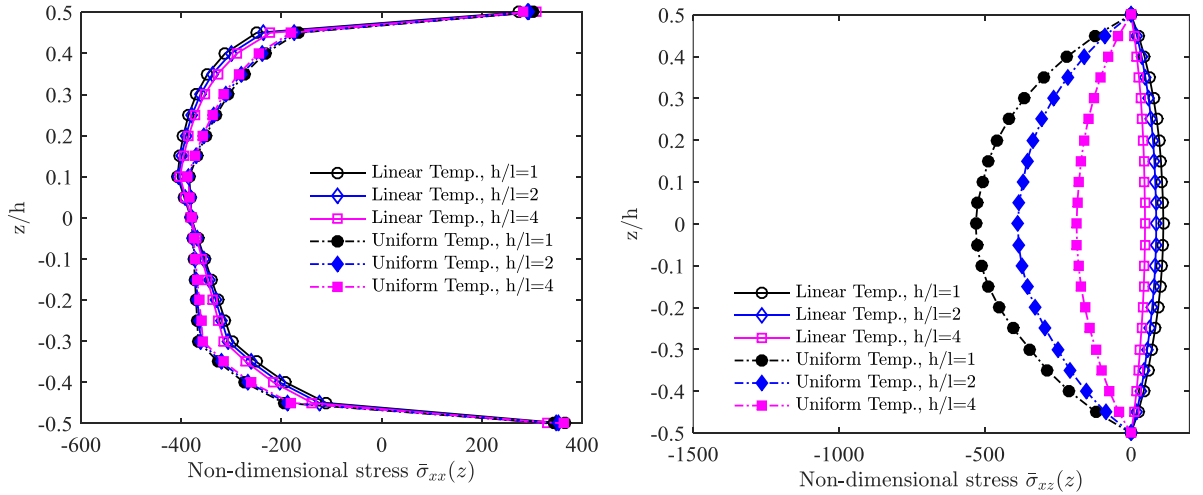
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a. FG-core sandwich plates

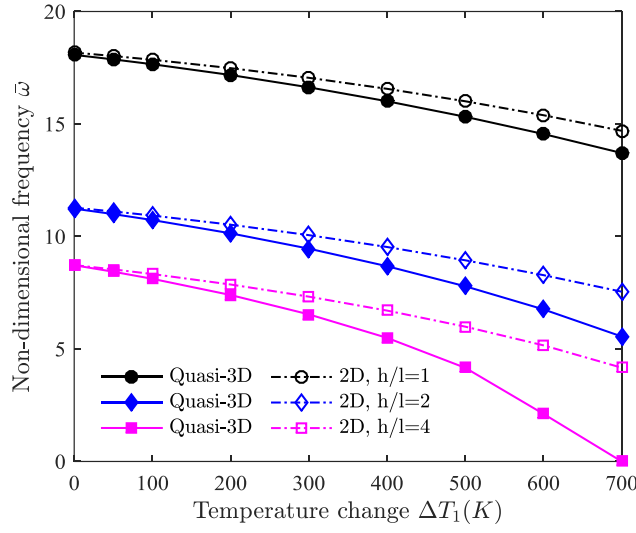


b. Ceramic-core sandwich plates

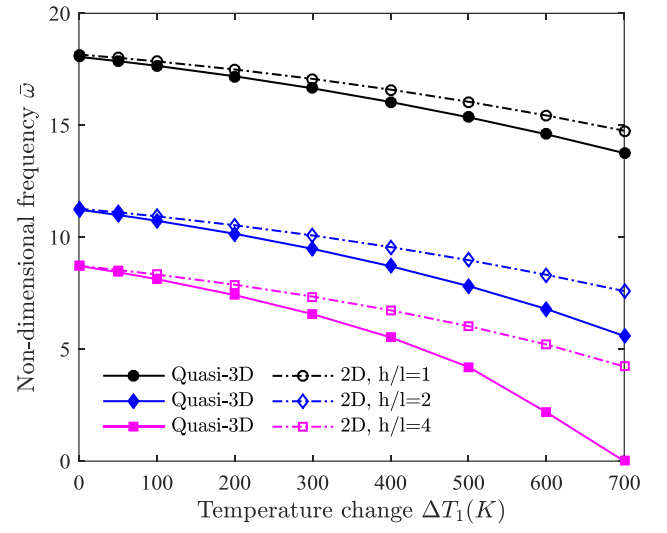


c. Metal-core sandwich plates

Fig. 6: Distribution of non-dimensional stresses of (1-2-2)  $\text{Si}_3\text{N}_4/\text{SUS304}$  microplates through the thickness for various material length scale ratios ( $p=0.2$ ,  $\Delta T_1 = 300\text{K}$ ).

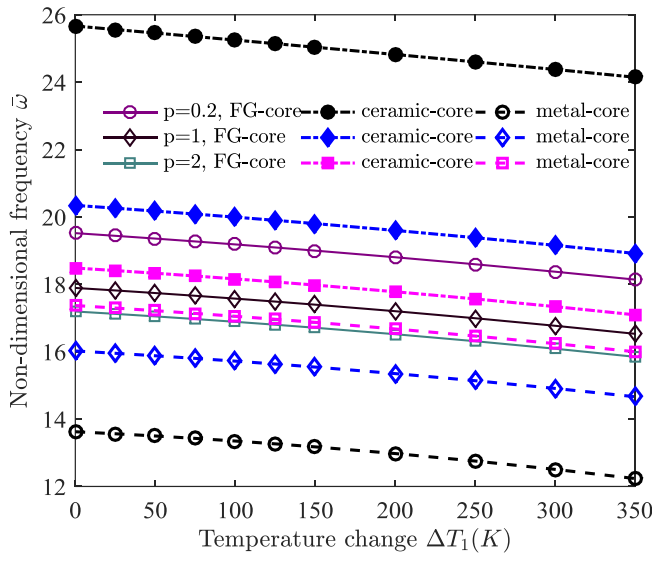


a. Uniform temperature

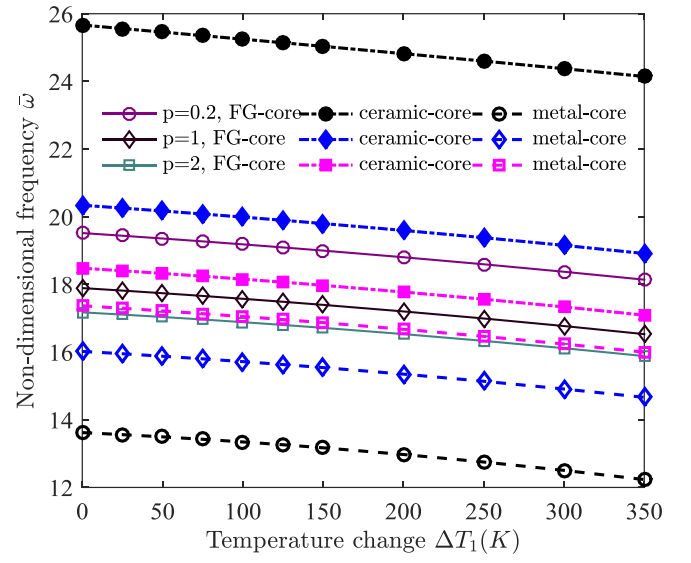


b. Linear temperature

Fig. 7: Non-dimensional frequencies of  $\text{Si}_3\text{N}_4/\text{SUS304}$  microplates for various material length scale ratios under uniform and linear temperatures.



a. Uniform



b. Linear

Fig. 8: Non-dimensional frequencies of (1-1-1)  $\text{Si}_3\text{N}_4/\text{SUS304}$  microplates for various power-law index under uniform and linear temperature ( $h/l=1$ ).

Table 1: Material properties of FG plates for mechanical and temperature-independent (TID) thermal analysis.

Material		Material properties		
		E (GPa)	$\alpha$ (1/K)	$\nu$
Metal:	Ti-6Al-4V	66.2	10.3e-6	1/3
	Al	70	-	0.3
	Mat <sub>1</sub>	14.4	-	0.38
Ceramic:	ZrO <sub>2</sub>	117	7.11e-6	1/3
	Al <sub>2</sub> O <sub>3</sub>	380	-	0.3
	Mat <sub>2</sub>	1.44	-	0.38

Table 2: Temperature-dependent coefficients of Si<sub>3</sub>N<sub>4</sub>/SUS304 plates.

Materials	Proprieties	P <sub>0</sub>	P <sub>-1</sub>	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>
Si <sub>3</sub> N <sub>4</sub>	E (Pa)	348.43e+9	0.0	-3.070e-4	2.160e-7	-8.946e-11
	$\alpha$ (1/K)	5.8723e-6	0.0	9.095e-4	0.0	0.0
	$\kappa$ (W/mK)	13.723	0.0	-1.032e-3	5.466e-7	-7.876e-11
	$\nu$	0.24	0.0	0.0	0.0	0.0
	$\rho$ (kg/m <sup>3</sup> )	2370	0.0	0.0	0.0	0.0
SUS304	E (Pa)	201.04e+9	0.0	3.079e-4	-6.534e-7	0.0
	$\alpha$ (1/K)	12.330e-6	0.0	8.086e-4	0.0	0.0
	$\kappa$ (W/mK)	15.379	0.0	-1.264e-3	2.092e-6	-7.223e-10
	$\nu$	0.3262	0.0	-2.002e-4	3.797e-7	0.0
	$\rho$ (kg/m <sup>3</sup> )	8166	0.0	0.0	0.0	0.0

Table 3: Non-dimensional deflections and stresses of Al/Al<sub>2</sub>O<sub>3</sub> microplates under sinusoidal loads.

a/h	h/l	Theory	$\bar{w}(a/2, b/2, 0)$			$\bar{\sigma}_{xx}(a/2, b/2, h/2)$		
			p=0	1	10	p=0	1	10
5	1	TSDT [38]	0.0569	0.0989	0.2077	0.2321	0.3162	0.6655
		SSDT [25]	0.0588	0.1017	0.2158	0.1800	0.2437	0.5189
		Present TSDT	0.0568	0.0985	0.2099	0.1841	0.2483	0.5294
		Present SSDT	0.0570	0.0988	0.2108	0.1844	0.2488	0.5289
		Quasi-3D (TSDT) [38]	0.0601	0.1031	0.2215	0.1878	0.2603	0.5232
		Present quasi-3D (TSDT)	0.0587	0.1008	0.2171	0.1858	0.2562	0.5228
	2	Present quasi-3D (SSDT)	0.0589	0.1012	0.2186	0.1968	0.2725	0.5531
		TSDT [38]	0.1432	0.2591	0.5113	0.5711	0.8185	1.5491
		Present TSDT	0.1502	0.2715	0.5352	0.4729	0.6702	1.2939
		Present SSDT	0.1506	0.2721	0.5387	0.4740	0.6720	1.2953
		Quasi-3D (TSDT) [38]	0.1564	0.2781	0.5623	0.4888	0.7022	1.3087
		Present quasi-3D (TSDT)	0.1533	0.2731	0.5468	0.4836	0.6930	1.2976
	4	Present quasi-3D (SSDT)	0.1536	0.2736	0.5505	0.5117	0.7364	1.3733
		TSDT [38]	0.2316	0.4363	0.8275	0.9086	1.3660	2.3797
		Present TSDT	0.2586	0.4881	0.9087	0.7891	1.1785	2.0694
		Present SSDT	0.2589	0.4886	0.9141	0.7912	1.1818	2.0759
		Quasi-3D (TSDT) [38]	0.2609	0.4827	0.9245	0.8156	1.2198	2.1200
		Present quasi-3D (TSDT)	0.2583	0.4784	0.9039	0.8105	1.2112	2.0981
	8	Present quasi-3D (SSDT)	0.2581	0.4781	0.9077	0.8543	1.2824	2.2117
		TSDT [38]	0.2740	0.5265	0.9865	1.0685	1.6427	2.7721
		Present TSDT	0.3171	0.6118	1.1243	0.9529	1.4609	2.4657
		Present SSDT	0.3172	0.6119	1.1275	0.9552	1.4648	2.4740
		Quasi-3D (TSDT) [38]	0.3133	0.5916	1.1064	0.9793	1.4953	2.5197
		Present quasi-3D (TSDT)	0.3123	0.5898	1.0963	0.9772	1.4917	2.5077
10	1	Present quasi-3D (SSDT)	0.3115	0.5886	1.0973	1.0276	1.5759	2.6329
		TSDT [38]	0.0530	0.0926	0.1968	0.4836	0.6541	1.3774
		SSDT [25]	0.0552	0.0959	0.2058	0.3749	0.5042	1.0733
		Present TSDT	0.0546	0.0951	0.2043	0.3770	0.5065	1.0787
		Present SSDT	0.0547	0.0951	0.2045	0.3771	0.5067	1.0785
		Quasi-3D (TSDT) [38]	0.0556	0.0960	0.2066	0.3787	0.5184	1.0785
	2	Present quasi-3D (TSDT)	0.0552	0.0954	0.2054	0.3778	0.5163	1.0785
		Present quasi-3D (SSDT)	0.0552	0.0954	0.2057	0.4018	0.5520	1.1411
		TSDT [38]	0.1283	0.2355	0.4577	1.1622	1.6561	3.1516
		Present TSDT	0.1401	0.2556	0.5022	0.9592	1.3542	2.6215
		Present SSDT	0.1402	0.2558	0.5031	0.9598	1.3550	2.6222
		Quasi-3D (TSDT) [38]	0.1419	0.2551	0.5050	0.9672	1.3775	2.6225
	4	Present quasi-3D (TSDT)	0.1410	0.2537	0.5008	0.9646	1.3728	2.6170
		Present quasi-3D (SSDT)	0.1410	0.2536	0.5015	1.0248	1.4665	2.7684
		TSDT [38]	0.1994	0.3837	0.6911	1.7954	2.6893	4.6805
		Present TSDT	0.2313	0.4437	0.7998	1.5683	2.3352	4.1032
		Present SSDT	0.2313	0.4438	0.8012	1.5694	2.3369	4.1066
		Quasi-3D (TSDT) [38]	0.2320	0.4354	0.7933	1.5815	2.3517	4.0981
	8	Present quasi-3D (TSDT)	0.2313	0.4343	0.7879	1.5790	2.3475	4.0871
		Present quasi-3D (SSDT)	0.2308	0.4335	0.7885	1.6746	2.5036	4.3180
		TSDT [38]	0.2314	0.4554	0.7946	2.0799	3.1877	5.3392
		Present TSDT	0.2766	0.5443	0.9455	1.8671	2.8552	4.7946
		Present SSDT	0.2766	0.5443	0.9464	1.8683	2.8572	4.7988
		Quasi-3D (TSDT) [38]	0.2757	0.5288	0.9265	1.8799	2.8567	4.7746
10	1	Present quasi-3D (TSDT)	0.2755	0.5284	0.9240	1.8789	2.8550	4.7687
		Present quasi-3D (SSDT)	0.2747	0.5271	0.9237	1.9908	3.0421	5.0322



Table 4: Deflections and stresses of Ti-6Al-4V/ZrO<sub>2</sub> plates under xy-sinusoidal and z-linear temperature.

p	Theory	z	$\bar{w}(a/2, b/2, z)$			$\bar{\sigma}_{xx}(a/2, b/2, h/2)$		
			FG	FG-core		FG	FG-core	
				1-2-2	1-1-1		1-2-2	1-1-1
0	SSDT [39]		-	0.5446	0.5698	-	-1.7508	-1.6663
	TSDT [39]		-	0.5446	0.5698	-	-1.7508	-1.6663
	Present SSDT		0.4803	0.5446	0.5698	-2.0797	-1.7508	-1.6663
	Present TSDT		0.4803	0.5446	0.5698	-2.0797	-1.7508	-1.6663
	Quasi-3D <sup>(a)</sup> (SSDT) [40]	z=h/2	-	0.5269	0.5526	-	-1.8544	-1.7284
		z=0	-	0.5144	0.5395	-	-	-
	Present quasi-3D <sup>(a)</sup> (SSDT)	z=h/2	0.4616	0.5269	0.5527	-2.2871	-1.8554	-1.7294
		z=0	0.4502	0.5144	0.5395	-	-	-
	Present quasi-3D <sup>(a)</sup> (TSDT)	z=h/2	0.4591	0.5227	0.5492	-1.9369	-1.4456	-1.3174
		z=0	0.4480	0.5103	0.5364	-	-	-
	Present quasi-3D <sup>(b)</sup> (SSDT)	z=h/2	0.4945	0.5673	0.5949	-2.8019	-2.0559	-1.8381
		z=0	0.4823	0.5540	0.5810	-	-	-
	Present quasi-3D <sup>(b)</sup> (TSDT)	z=h/2	0.4897	0.5587	0.5884	-2.0592	-1.1907	-0.9646
		z=0	0.4779	0.5455	0.5747	-	-	-
	SSDT [39]		-	0.5731	0.5788	-	-1.6745	-1.6646
	TSDT [39]		-	0.5731	0.5788	-	-1.6745	-1.6645
1	Present SSDT		0.5798	0.5731	0.5788	-1.5220	-1.6745	-1.6646
	Present TSDT		0.5798	0.5731	0.5788	-1.5220	-1.6745	-1.6645
	Quasi-3D <sup>(a)</sup> (SSDT) [40]	z=h/2	-	0.5561	0.5624	-	-1.7377	-1.7204
		z=0	-	0.5429	0.5490	-	-	-
	Present quasi-3D <sup>(a)</sup> (SSDT)	z=h/2	0.5597	0.5562	0.5624	-1.5307	-1.7388	-1.7215
		z=0	0.5461	0.5430	0.5491		-	-
	Present quasi-3D <sup>(a)</sup> (TSDT)	z=h/2	0.5571	0.5529	0.5593	-1.1158	-1.3257	-1.3097
		z=0	0.5439	0.5400	0.5464		-	-
	Present quasi-3D <sup>(b)</sup> (SSDT)	z=h/2	0.6006	0.5987	0.6055	-1.4865	-1.8563	-1.8267
		z=0	0.5861	0.5847	0.5913			
	Present quasi-3D <sup>(b)</sup> (TSDT)	z=h/2	0.5959	0.5926	0.5999	-0.6062	-0.9800	-0.9520
		z=0	0.5818	0.5789	0.5861	-	-	-
	SSDT [39]		-	0.5799	0.5807	-	-1.6732	-1.6638
	TSDT [39]		-	0.5799	0.5807	-	-1.6732	-1.6637
	Present SSDT		0.6247	0.5799	0.5807	-1.0700	-1.6732	-1.6638

Present TSDT		0.6248	0.5799	0.5807	-1.0699	-1.6732	-1.6637
Quasi-3D <sup>(a)</sup> (SSDT) [40]	z=h/2	-	0.5638	0.5641	-	-1.7318	-1.7156
	z=0	-	0.5505	0.5507	-	-	-
Present quasi-3D <sup>(a)</sup> (SSDT)	z=h/2	0.6004	0.5638	0.5641	-0.9094	-1.7329	-1.7167
	z=0	0.5853	0.5505	0.5507	-	-	-
Present quasi-3D <sup>(a)</sup> (TSDT)	z=h/2	0.5990	0.5607	0.5611	-0.4848	-1.3207	-1.3067
	z=0	0.5847	0.5477	0.5482	-	-	-
Present quasi-3D <sup>(b)</sup> (SSDT)	z=h/2	0.6419	0.6073	0.6071	-0.3865	-1.8475	-1.8182
	z=0	0.6258	0.5931	0.5929	-	-	-
Present quasi-3D <sup>(b)</sup> (TSDT)	z=h/2	0.6398	0.6016	0.6020	0.5162	-0.9718	-0.9469
	z=0	0.6246	0.5878	0.5881	-	-	-

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<sup>(a)</sup>: Results obtained with  $[Q_{ii}^{2D}, Q_{ij}^{2D}]$ ; <sup>(b)</sup>: Results obtained with  $[Q_{ii}^{3D}, Q_{ij}^{3D}]$

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Table 5: Non-dimensional fundamental frequencies  $\hat{\omega}$  of Al/Al<sub>2</sub>O<sub>3</sub> microplates.

a/h	h/l	Theory	p				
			0	0.5	1	5	10
5	1	TSDT [38]	12.9565	11.4813	10.5983	8.4762	7.8468
		Present TSDT	13.1645	11.6622	10.7482	8.5714	7.9417
		Quasi-3D (TSDT) [38]	12.4747	11.1097	10.2849	8.2027	7.5480
		Present quasi-3D (TSDT)	12.5865	11.2082	10.3732	8.2599	7.6030
	2	TSDT [38]	8.1759	7.1553	6.5542	5.3491	5.0233
		Present TSDT	8.3177	7.2634	6.6505	5.4479	5.1267
		Quasi-3D (TSDT) [38]	7.7588	6.8328	6.2857	5.1083	4.7628
		Present quasi-3D (TSDT)	7.8212	6.8784	6.3301	5.1591	4.8150
	4	TSDT [38]	6.4393	5.5604	5.0553	4.1718	3.9619
		Present TSDT	6.5027	5.6107	5.0984	4.2352	4.0303
		Quasi-3D (TSDT) [38]	6.0161	5.2220	4.7737	3.9453	3.7260
		Present quasi-3D (TSDT)	6.0397	5.2428	4.7924	3.9796	3.7606
	8	TSDT [38]	5.9205	5.0878	4.6015	3.8080	3.6331
		Present TSDT	5.9413	5.1018	4.6169	3.8310	3.6582
		Quasi-3D (TSDT) [38]	5.4951	4.7406	4.3143	3.5943	3.4113
		Present quasi-3D (TSDT)	5.4983	4.7429	4.3193	3.6038	3.4226
10	1	TSDT [38]	13.6329	12.0824	11.1415	8.9022	8.2367
		Present TSDT	13.7017	12.1439	11.1933	8.9284	8.2615
		Quasi-3D (TSDT) [38]	13.2922	11.8250	10.9199	8.7034	8.0198
		Present quasi-3D (TSDT)	13.3344	11.8606	10.9617	8.7279	8.0421
	2	TSDT [38]	8.7651	7.6413	6.9954	5.7361	5.3998
		Present TSDT	8.8098	7.6774	7.0202	5.7651	5.4330
		Quasi-3D (TSDT) [38]	8.3187	7.3006	6.7130	5.4839	5.1335
		Present quasi-3D (TSDT)	8.3415	7.3148	6.7225	5.5062	5.1525
	4	TSDT [38]	7.0338	6.0428	5.4762	4.5989	4.3928
		Present TSDT	7.0538	6.0534	5.4902	4.6190	4.4186
		Quasi-3D (TSDT) [38]	6.5011	5.6087	5.1293	4.3069	4.0903
		Present quasi-3D (TSDT)	6.5153	5.6282	5.1396	4.3282	4.1092
	8	TSDT [38]	6.5258	5.5750	5.0315	4.2613	4.0974
		Present TSDT	6.5333	5.5692	5.0309	4.2690	4.1078
		Quasi-3D (TSDT) [38]	5.9678	5.1146	4.6442	3.9723	3.7809
		Present quasi-3D (TSDT)	5.9704	5.1197	4.6597	3.9724	3.7955

Table 6: Non-dimensional fundamental frequencies  $\bar{\omega}$  of  $\text{Si}_3\text{N}_4/\text{SUS304}$  plates ( $\Delta T = 400K$  and  $a/h = 10$ ).

p	Theory	TID solution			TD solution		
		a/b = 1	a/b = 2	a/b = 3	a/b = 1	a/b = 2	a/b = 3
0	CPT [41]	11.417	30.671	61.696	10.512	29.123	59.048
	FSDT [41]	11.095	29.001	56.078	10.187	27.485	53.584
	TSDT [41]	11.033	28.696	55.121	10.124	27.186	52.652
	Present TSDT	11.170	28.966	55.547	10.302	27.527	53.187
	Present quasi-3D (TSDT) <sup>(+)</sup>	11.099	28.884	55.571	10.204	27.399	53.139
	Present quasi-3D (TSDT) <sup>(++)</sup>	10.084	28.139	55.010	8.858	26.459	52.442
0.5	CPT [41]	7.448	20.776	42.166	6.642	19.431	39.870
	FSDT [41]	7.217	19.622	38.330	6.405	18.306	36.174
	TSDT [41]	7.174	19.417	37.694	6.362	18.108	35.563
	Present TSDT	7.264	19.571	37.886	6.479	18.299	35.805
	Present quasi-3D (TSDT) <sup>(+)</sup>	7.201	19.500	37.905	6.392	18.191	35.769
	Present quasi-3D (TSDT) <sup>(++)</sup>	6.163	18.781	37.371	4.976	17.295	35.115
1	CPT [41]	6.388	18.142	37.008	5.619	16.863	34.819
	FSDT [41]	6.176	17.099	33.549	5.401	15.848	31.491
	TSDT [41]	6.135	16.909	32.959	5.359	15.662	30.923
	Present TSDT	6.204	17.011	33.057	5.445	15.785	31.039
	Present quasi-3D (TSDT) <sup>(+)</sup>	6.139	16.932	33.042	5.357	15.673	30.975
	Present quasi-3D (TSDT) <sup>(++)</sup>	5.088	16.231	32.526	3.902	14.804	30.351
5	CPT [41]	4.969	14.606	30.070	4.266	13.433	28.036
	FSDT [41]	4.776	13.687	27.034	4.063	12.534	25.108
	TSDT [41]	4.732	13.491	26.437	4.015	12.336	24.514
	Present TSDT	4.841	13.675	26.689	4.156	12.556	24.810
	Present quasi-3D (TSDT) <sup>(+)</sup>	4.757	13.551	26.569	4.042	12.394	24.637
	Present quasi-3D (TSDT) <sup>(++)</sup>	3.695	12.898	26.104	2.539	11.608	24.094
<sup>(+)</sup> : Exclusion of thermal thickness stress ( $\sigma_{zz}^T = 0$ ), <sup>(++)</sup> : Inclusion of thermal thickness stress ( $\sigma_{zz}^T \neq 0$ )							

Table 7: Non-dimensional critical buckling loads of Mat<sub>1</sub>/Mat<sub>2</sub> microplates.

h/l	Theory	$a/h = 5$			$a/h = 10$			$a/h = 20$		
		p=0	1	10	p=0	1	10	0	1	10
1	RPT [26]	82.694	43.809	15.952	88.542	46.537	16.603	90.180	47.291	16.779
	RPT [28]	78.968	42.039	15.407	87.378	45.998	16.443	89.872	47.149	16.738
	Present TSDT	85.767	45.262	16.272	89.332	46.908	16.605	89.047	47.027	16.319
	Quasi-3D (RPT) [28]	73.693	39.887	14.529	85.604	45.822	16.482	89.402	47.660	17.083
	Present quasi-3D (TSDT)	86.070	45.969	16.576	89.332	47.312	17.009	89.047	47.027	16.319
2	Present TSDT	33.372	16.562	6.517	35.961	17.587	6.885	36.424	17.848	6.540
	Present quasi-3D (TSDT)	34.028	17.420	6.820	36.163	18.394	7.289	36.443	18.665	7.352
4	Present TSDT	19.952	9.321	3.841	22.535	10.212	4.353	23.070	10.545	4.484
	Present quasi-3D (TSDT)	20.937	10.280	4.194	22.939	11.020	4.757	23.474	10.949	4.888
8	Present TSDT	16.504	7.476	3.057	19.197	8.439	3.692	20.020	8.707	3.858
	Present quasi-3D (TSDT)	17.627	8.473	3.423	19.601	9.247	4.146	20.020	9.313	4.262
Classical	RPT [26]	15.332	6.861	2.767	18.075	7.828	3.497	18.924	8.114	3.745
	RPT [28]	15.332	6.861	2.770	18.076	7.828	3.498	18.924	8.114	3.745
	Present TSDT	15.332	6.861	2.767	18.075	7.828	3.497	18.924	8.113	3.745
	Quasi-3D (RPT) [28]	15.363	7.391	3.012	18.156	8.540	3.892	18.968	8.864	4.185
	Present quasi-3D (TSDT)	16.527	7.876	3.148	18.489	8.673	3.930	19.037	8.891	4.188

Table 8: Critical buckling temperatures  $\Delta T_{cr} [K]$  of Al/Al<sub>2</sub>O<sub>3</sub> microplates.

p	h/l	Theory	Uniform temperature			Linear temperature		
			a/h=100	20	10	100	20	10
0	Classical	HSDT [42]	17.080	421.530	1618.680	24.170	833.070	3227.360
		HSDT [43]	17.089	421.540	1618.750	24.179	833.079	3227.510
		CBT [31]	17.099	-	-	24.198	-	-
		Present TSDT	17.089	421.535	1618.689	24.177	833.073	3227.377
		Present quasi-3D (TSDT)	17.083	423.290	1644.410	24.182	836.576	3278.829
	2	CBT [31]	35.053	-	-	60.107	-	-
		Present TSDT	35.042	870.790	3419.221	60.084	1731.582	6828.442
		Present quasi-3D (TSDT)	35.042	871.571	3431.200	60.086	1733.152	6852.410
	1	CBT [31]	88.916	-	-	167.831	-	-
		Present TSDT	88.902	2215.320	8772.104	167.806	4420.639	17534.209
		Present quasi-3D (TSDT)	88.891	2215.610	8776.800	167.798	4421.229	17543.610
1	Classical	HSDT [42]	7.940	196.260	758.390	5.510	358.710	1412.960
		HSDT [43]	7.940	196.267	758.424	5.513	358.715	1413.020
		CBT [31]	7.944	-	-	5.521	-	-
		Present TSDT	7.939	196.266	758.398	5.512	358.713	1412.975
		Present quasi-3D (TSDT)	8.174	202.797	790.829	5.955	370.963	1473.811
	2	CBT [31]	17.852	-	-	24.104	-	-
		Present TSDT	17.846	443.874	1747.416	24.093	823.094	3267.841
		Present quasi-3D (TSDT)	18.082	450.093	1775.366	24.541	834.758	3320.262
	1	CBT [31]	47.577	-	-	79.852	-	-
		Present TSDT	47.571	1185.721	4699.088	79.839	2214.401	8803.603
		Present quasi-3D (TSDT)	47.803	1191.776	4724.691	80.285	2225.760	8851.616
5	Classical	HSDT [42]	7.260	178.530	679.310	3.891	298.700	1160.680
		HSDT [43]	7.260	178.516	679.039	3.890	298.672	1160.220
		CBT [31]	7.266	-	-	3.900	-	-
		Present TSDT	7.258	178.535	679.312	3.889	298.704	1160.689
		Present quasi-3D (TSDT)	7.547	186.246	713.734	4.394	311.968	1219.936
	2	CBT [31]	14.692	-	-	16.683	-	-
		Present TSDT	14.687	365.354	1438.925	16.674	620.276	2468.204
		Present quasi-3D (TSDT)	14.971	372.589	1466.682	17.181	632.736	2515.987
	1	CBT [31]	36.971	-	-	55.031	-	-
		Present TSDT	36.967	922.164	3663.791	55.023	1578.709	6297.849
		Present quasi-3D (TSDT)	37.256	929.248	3689.120	55.530	1590.906	6341.446

Table 9: Non-dimensional deflections of FG and (1-2-2) Si<sub>3</sub>N<sub>4</sub>/SUS304 sandwich microplates under uniform temperature ( $a/h = 5$ ).

h/l	p	Theory	FG		FG-core		Ceramic-core		Metal-core	
			$\Delta T_1=300$	900	300	900	300	900	300	900
1	0	SSDT	0	0	-0.381	-1.075	0	0	0	0
		Quasi-3D (SSDT)	0	0	-0.435	-2.713	0	0	0	0
	1	SSDT	-0.522	-2.276	-0.653	-2.449	0.170	0.666	<b>-0.202</b>	<b>-0.920</b>
		Quasi-3D (SSDT)	-0.585	-4.523	-0.696	-5.036	0.159	1.094	<b>-0.238</b>	<b>-2.218</b>
	5	SSDT	-0.448	-2.482	-0.769	-3.086	0.250	0.996	<b>-0.238</b>	<b>-0.881</b>
		Quasi-3D (SSDT)	-0.581	-6.179	-0.847	-6.795	0.231	1.910	<b>-0.251</b>	<b>-2.021</b>
	5	SSDT	0	0	-1.921	-5.863	0	0	0	0
		Quasi-3D (SSDT)	0	0	-2.978	-17.339	0	0	0	0
	1	SSDT	-2.437	-10.724	-3.171	-12.921	0.862	3.658	<b>-0.847</b>	<b>-3.302</b>
		Quasi-3D (SSDT)	-3.530	-23.143	-4.531	-28.813	1.147	7.160	<b>-1.286</b>	<b>-8.828</b>
	5	SSDT	-1.957	-9.930	-3.596	-15.463	1.280	5.948	<b>-1.002</b>	<b>-3.296</b>
		Quasi-3D (SSDT)	-3.057	-25.040	-5.167	-34.746	1.685	12.749	<b>-1.470</b>	<b>-8.910</b>
Classical	0	SSDT	0	0	-2.321	-7.219	0	0	0	0
		Quasi-3D (SSDT)	0	0	-3.617	-21.072	0	0	0	0
	1	SSDT	-2.896	-12.774	-3.799	-15.789	1.042	4.508	<b>-0.985</b>	<b>-3.744</b>
		Quasi-3D (SSDT)	-4.221	-27.238	-5.457	-34.458	1.397	8.759	<b>-1.510</b>	<b>-9.975</b>
	5	SSDT	-2.296	-11.478	-4.274	-18.685	1.551	7.490	<b>-1.167</b>	<b>-3.754</b>
		Quasi-3D (SSDT)	-3.598	-28.485	-6.171	-40.844	2.053	15.790	<b>-1.737</b>	<b>-10.190</b>

Table 10: Non-dimensional deflections of FG and (1-2-2) Si<sub>3</sub>N<sub>4</sub>/SUS304 sandwich microplates under linear temperature ( $a/h = 5$ ).

h/l	p	Theory	FG		FG-core		Ceramic-core		Metal-core		
			$\Delta T_1=0$	300	0	300	0	300	0	300	
Classical	1	0	SSDT	0.119	0.170	0.155	-0.165	0.119	0.170	<b>0.282</b>	<b>0.403</b>
			Quasi-3D (SSDT)	0.061	0.092	0.087	-0.305	0.061	0.091	<b>0.148</b>	<b>0.251</b>
		1	SSDT	0.189	-0.254	0.177	-0.404	0.181	0.419	<b>0.199</b>	<b>0.085</b>
			Quasi-3D (SSDT)	0.104	-0.422	0.100	-0.541	0.099	0.306	<b>0.103</b>	<b>-0.071</b>
		5	SSDT	0.237	-0.108	0.189	-0.502	0.211	0.537	<b>0.158</b>	<b>-0.010</b>
			Quasi-3D (SSDT)	0.126	-0.372	0.109	-0.675	0.113	0.404	<b>0.083</b>	<b>-0.119</b>
	5	0	SSDT	0.571	0.815	0.781	-0.833	0.571	0.816	<b>1.263</b>	<b>1.779</b>
			Quasi-3D (SSDT)	0.553	0.801	0.809	-1.816	0.553	0.802	<b>1.230</b>	<b>1.855</b>
		1	SSDT	0.886	-1.183	0.864	-1.955	0.919	2.128	<b>0.845</b>	<b>0.361</b>
			Quasi-3D (SSDT)	0.886	-2.228	0.885	-3.237	0.938	2.477	<b>0.809</b>	<b>-0.074</b>
		5	SSDT	1.042	-0.476	0.888	-2.336	1.085	2.761	<b>0.677</b>	<b>-0.029</b>
			Quasi-3D (SSDT)	1.019	-1.526	0.910	-3.819	1.109	3.263	<b>0.654</b>	<b>-0.496</b>
	0	SSDT	0.683	0.975	0.944	-1.006	0.683	0.975	<b>1.487</b>	<b>2.089</b>	
			Quasi-3D (SSDT)	0.676	0.979	0.993	-2.194	0.676	0.979	<b>1.478</b>	<b>2.214</b>
		1	SSDT	1.055	-1.405	1.036	-2.340	1.112	2.575	<b>0.986</b>	<b>0.422</b>
				Quasi-3D (SSDT)	1.075	-2.646	1.079	-3.883	1.152	3.028	<b>0.971</b>
		5	SSDT	1.224	-0.560	1.058	-2.774	1.316	3.347	<b>0.792</b>	<b>-0.030</b>
				Quasi-3D (SSDT)	1.224	-1.769	1.102	-4.544	1.363	3.988	<b>0.787</b>



Table 11: Non-dimensional fundamental frequencies  $\bar{\omega}$  of FG-core  $\text{Si}_3\text{N}_4/\text{SUS304}$  microplates under uniform and linear temperatures ( $a/h = 10$ ).

Temp.	h/l	$\Delta T_1$	Theory	1-2-2			1-1-1		
				0	1	5	0	1	5
Uniform	1	300	TSDT	22.4338	18.6189	16.7630	19.7294	17.1845	15.8297
			Quasi-3D (TSDT)	22.0117	18.2000	16.3504	19.3065	16.7698	15.4215
	600	TSDT		20.9876	17.0665	15.1717	18.2403	15.5951	14.2002
			Quasi-3D (TSDT)	20.5158	16.5776	14.6816	17.7495	15.1040	13.7117
	2	300	TSDT	13.5455	11.2868	10.2184	11.9329	10.4568	9.6866
			Quasi-3D (TSDT)	12.9872	10.7364	9.6809	11.3723	9.9151	9.1571
	600	TSDT		11.9747	9.7006	8.6644	10.3869	8.8846	8.1331
			Quasi-3D (TSDT)	11.2999	8.9945	7.9589	9.6762	8.1757	7.4303
	4	300	TSDT	10.1607	8.4986	7.7354	8.9662	7.9025	7.3584
			Quasi-3D (TSDT)	9.4653	7.8201	7.0824	8.2688	7.2412	6.7212
	600	TSDT		8.2989	6.6680	5.9843	7.1708	6.1178	5.6288
			Quasi-3D (TSDT)	7.3687	5.6853	5.0148	6.1802	5.1373	4.6710
	8	300	TSDT	9.1121	7.6342	6.9647	8.0472	7.1099	6.6341
			Quasi-3D (TSDT)	8.3549	6.8993	6.2634	7.2881	6.3978	5.9539
	600	TSDT		7.0791	5.6485	5.0791	6.0979	5.1835	4.7769
			Quasi-3D (TSDT)	5.9950	4.4900	3.9445	4.9322	4.0308	3.6616
Linear	1	300	TSDT	22.4592	18.6501	16.7943	19.7607	17.2157	15.8591
			Quasi-3D (TSDT)	22.0338	18.2277	16.3784	19.3341	16.7976	15.4477
	600	TSDT		21.0472	17.1390	15.2447	18.3125	15.6678	14.2700
			Quasi-3D (TSDT)	20.5644	16.6387	14.7433	17.8097	15.1653	13.7704
	2	300	TSDT	13.5738	11.3207	10.2520	11.9669	10.4904	9.7183
			Quasi-3D (TSDT)	13.0113	10.7663	9.7106	11.4020	9.9447	9.1850
	600	TSDT		12.0339	9.7689	8.7311	10.4547	8.9517	8.1975
			Quasi-3D (TSDT)	11.3435	9.0473	8.0102	9.7273	8.2272	7.4797
	4	300	TSDT	10.1934	8.5375	7.7734	9.0051	7.9406	7.3941
			Quasi-3D (TSDT)	9.4937	7.8553	7.1168	8.3036	7.2757	6.7535
	600	TSDT		8.3668	6.7443	6.0566	7.2462	6.1911	5.6986
			Quasi-3D (TSDT)	7.4182	5.7447	5.0708	6.2367	5.1941	4.7252
	8	300	TSDT	9.1471	7.6756	7.0049	8.0887	7.1503	6.6718
			Quasi-3D (TSDT)	8.3858	6.9374	6.3005	7.3259	6.4350	5.9887
	600	TSDT		7.1531	5.7309	5.1562	6.1793	5.2621	4.8510
			Quasi-3D (TSDT)	6.0504	4.5573	4.0072	4.9955	4.0947	3.7225

Table 12: Non-dimensional fundamental frequencies  $\bar{\omega}$  of ceramic-core Si<sub>3</sub>N<sub>4</sub>/SUS304 microplates under uniform and linear temperatures ( $a/h = 10$ ).

Temp.	h/l	$\Delta T_1$	Theory	1-2-2			1-1-1		
				0	1	5	0	1	5
Uniform	1	300	TSDT	28.8114	20.2316	16.9970	28.8114	19.5962	16.2481
			Quasi-3D (TSDT)	28.3804	19.7974	16.5421	28.3804	19.1574	15.7836
	600	300	TSDT	27.3966	18.6788	15.3814	27.3966	18.0212	14.5935
			Quasi-3D (TSDT)	26.9733	18.1707	14.8047	26.9733	17.4994	13.9915
	2	300	TSDT	17.5032	12.1752	10.2190	17.5032	11.7825	9.7630
			Quasi-3D (TSDT)	16.9854	11.5797	9.5750	16.9854	11.1744	9.0978
	600	300	TSDT	15.8513	10.5199	8.5980	15.8513	10.1166	8.1234
			Quasi-3D (TSDT)	15.3187	9.7509	7.6754	15.3187	9.3138	7.1393
	4	300	TSDT	13.2177	9.0990	7.6294	13.2177	8.7970	7.2846
			Quasi-3D (TSDT)	12.6045	8.3414	6.7943	12.6045	8.0184	6.4155
	600	300	TSDT	11.2134	7.1404	5.7616	11.2134	6.8299	5.4041
			Quasi-3D (TSDT)	10.5380	6.0225	4.3372	10.5380	5.6433	3.8379
	8	300	TSDT	11.8927	8.1443	6.8256	11.8927	7.8701	6.5153
			Quasi-3D (TSDT)	11.2411	7.3107	5.8976	11.2411	7.0106	5.5456
	600	300	TSDT	9.6960	5.9987	4.7925	9.6960	5.7144	4.4689
			Quasi-3D (TSDT)	8.9467	4.6440	2.9635	8.9467	4.2567	2.3797
Linear	1	300	TSDT	28.8116	20.2191	16.9825	28.8116	19.5927	16.2436
			Quasi-3D (TSDT)	28.3806	19.7860	16.5289	28.3806	19.1542	15.7797
	600	300	TSDT	27.3956	18.6554	15.3549	27.3956	18.0177	14.5891
			Quasi-3D (TSDT)	26.9723	18.1505	14.7823	26.9723	17.4967	13.9887
	2	300	TSDT	17.5028	12.1621	10.2047	17.5028	11.7792	9.7591
			Quasi-3D (TSDT)	16.9849	11.5679	9.5621	16.9849	11.1716	9.0946
	600	300	TSDT	15.8500	10.4987	8.5764	15.8500	10.1138	8.1199
			Quasi-3D (TSDT)	15.3174	9.7342	7.6595	15.3174	9.3121	7.1382
	4	300	TSDT	13.2169	9.0842	7.6135	13.2169	8.7935	7.2806
			Quasi-3D (TSDT)	12.6036	8.3278	6.7797	12.6036	8.0153	6.4122
	600	300	TSDT	11.2117	7.1170	5.7392	11.2117	6.8270	5.4005
			Quasi-3D (TSDT)	10.5363	6.0039	4.3210	10.5363	5.6419	3.8380
	8	300	TSDT	11.8916	8.1285	6.8088	11.8916	7.8665	6.5112
			Quasi-3D (TSDT)	11.2400	7.2959	5.8817	11.2400	7.0073	5.5422
	600	300	TSDT	9.6941	5.9733	4.7686	9.6941	5.7114	4.4652
			Quasi-3D (TSDT)	8.9447	4.6226	2.9440	8.9447	4.2553	2.3807

**Table 13: Non-dimensional fundamental frequencies  $\bar{\omega}$  of metal-core Si<sub>3</sub>N<sub>4</sub>/SUS304 microplates under uniform and linear temperatures ( $a/h=10$ ).**

Temp.	h/l	$\Delta T_1$	Theory	1-2-2			1-1-1		
				0	1	5	0	1	5
Uniform	1	300	TSDT	11.9407	14.9082	17.4338	11.9407	15.2919	18.2617
			Quasi-3D (TSDT)	11.4623	14.5204	17.0666	11.4623	14.9104	17.8979
	600	300	TSDT	9.9850	13.1901	15.8828	9.9850	13.5931	16.7388
			Quasi-3D (TSDT)	9.3046	12.7394	15.4927	9.3046	13.1591	16.3631
	2	300	TSDT	7.3231	9.2076	10.7613	7.3231	9.4488	11.2728
			Quasi-3D (TSDT)	6.6535	8.7145	10.3174	6.6535	8.9689	10.8405
	600	300	TSDT	5.5929	7.6072	9.2621	5.5929	7.8549	9.7792
			Quasi-3D (TSDT)	4.4629	6.9703	8.7523	4.4629	7.2510	9.2999
	4	300	TSDT	5.5729	7.0523	8.2421	5.5729	7.2399	8.6342
			Quasi-3D (TSDT)	4.7358	6.4750	7.7334	4.7358	6.6815	8.1427
	600	300	TSDT	3.7079	5.2892	6.5676	3.7079	5.4778	6.9544
			Quasi-3D (TSDT)	1.7897	4.4490	5.9331	1.7897	4.6918	6.3666
	8	300	TSDT	5.0284	6.3771	7.4544	5.0284	6.5472	7.8088
			Quasi-3D (TSDT)	4.1170	5.7714	6.9264	4.1170	5.9635	7.3006
	600	300	TSDT	3.0349	4.4826	5.6535	3.0349	4.6516	5.9985
			Quasi-3D (TSDT)	0.0000	3.5267	4.9612	0.0000	3.7666	5.3631
Linear	1	300	TSDT	11.9363	14.9136	17.4425	11.9363	15.2905	18.2615
			Quasi-3D (TSDT)	11.4584	14.5251	17.0747	11.4584	14.9091	17.8977
	600	300	TSDT	9.9800	13.2033	15.8999	9.9800	13.5913	16.7378
			Quasi-3D (TSDT)	9.3015	12.7506	15.5082	9.3015	13.1575	16.3622
	2	300	TSDT	7.3194	9.2132	10.7694	7.3194	9.4475	11.2723
			Quasi-3D (TSDT)	6.6504	8.7194	10.3250	6.6504	8.9678	10.8400
	600	300	TSDT	5.5890	7.6177	9.2730	5.5890	7.8537	9.7782
			Quasi-3D (TSDT)	4.4620	6.9779	8.7610	4.4620	7.2500	9.2990
	4	300	TSDT	5.5694	7.0585	8.2507	5.5694	7.2386	8.6335
			Quasi-3D (TSDT)	4.7329	6.4805	7.7416	4.7329	6.6803	8.1420
	600	300	TSDT	3.7042	5.2992	6.5761	3.7042	5.4767	6.9534
			Quasi-3D (TSDT)	1.7910	4.4556	5.9389	1.7910	4.6910	6.3656
	8	300	TSDT	5.0249	6.3835	7.4633	5.0249	6.5459	7.8080
			Quasi-3D (TSDT)	4.1141	5.7773	6.9349	4.1141	5.9622	7.2998
	600	300	TSDT	3.0312	4.4925	5.6612	3.0312	4.6506	5.9974
			Quasi-3D (TSDT)	0.0000	3.5333	4.9659	0.0000	3.7659	5.3620

Table 14: Critical buckling temperatures  $\Delta T_{cr} [K]$  of (1-2-2)  $\text{Si}_3\text{N}_4/\text{SUS304}$  microplates ( $a/h = 20$ ).

Temp. pattern	h/l	Theory	FG core			Ceramic core			Metal core		
			p=0.2	1	5	0.2	1	5	0.2	1	5
Uniform	1	TSDT	1053.28	1005.92	997.92	1095.52	1018.72	979.36	<b>724.78</b>	<b>835.04</b>	<b>977.28</b>
		Quasi-3D (TSDT)	1075.20	1033.60	1052.80	1096.96	1020.80	982.56	<b>725.76</b>	<b>836.32</b>	<b>978.56</b>
	2	TSDT	512.80	481.76	462.56	568.48	497.12	448.48	<b>364.96</b>	<b>422.56</b>	<b>483.36</b>
		Quasi-3D (TSDT)	515.20	486.40	467.20	569.60	498.08	449.60	<b>365.28</b>	<b>423.20</b>	<b>484.16</b>
	4	TSDT	336.80	316.00	304.16	380.64	322.40	284.64	<b>244.32</b>	<b>288.48</b>	<b>332.32</b>
		Quasi-3D (TSDT)	339.20	320.00	307.20	381.92	323.52	285.92	<b>244.96</b>	<b>289.44</b>	<b>333.28</b>
	8	TSDT	287.20	269.60	260.64	327.20	273.12	239.20	<b>211.04</b>	<b>251.68</b>	<b>290.40</b>
		Quasi-3D (TSDT)	291.20	272.00	262.40	328.64	274.40	240.48	<b>212.00</b>	<b>252.64</b>	<b>291.68</b>
Linear	Classical	TSDT	270.24	253.92	245.60	308.64	256.16	223.52	<b>199.52</b>	<b>238.88</b>	<b>276.00</b>
		Quasi-3D (TSDT)	272.00	256.00	249.60	310.08	257.44	224.80	<b>200.64</b>	<b>240.00</b>	<b>277.44</b>
	1	TSDT	1041.76	993.76	980.64	1095.84	1021.60	986.08	<b>724.84</b>	<b>833.12</b>	<b>969.82</b>
		Quasi-3D (TSDT)	1064.00	1018.72	1020.96	1097.44	1024.00	989.76	<b>725.44</b>	<b>834.08</b>	<b>970.88</b>
	2	TSDT	515.36	484.64	465.44	567.84	496.48	448.16	<b>364.96</b>	<b>422.88</b>	<b>483.36</b>
		Quasi-3D (TSDT)	520.00	489.60	470.72	568.80	497.44	449.12	<b>365.44</b>	<b>423.36</b>	<b>484.16</b>
	4	TSDT	340.32	319.84	308.00	379.68	321.12	283.68	<b>244.32</b>	<b>289.12</b>	<b>332.96</b>
		Quasi-3D (TSDT)	344.16	324.00	312.48	380.96	322.24	284.64	<b>244.96</b>	<b>289.92</b>	<b>333.92</b>
	8	TSDT	291.04	273.76	264.48	326.24	271.84	237.92	<b>211.04</b>	<b>252.00</b>	<b>291.36</b>
		Quasi-3D (TSDT)	294.88	277.92	268.96	327.52	273.12	239.04	<b>212.00</b>	<b>253.12</b>	<b>292.48</b>
	Classical	TSDT	273.76	257.76	249.44	307.68	254.88	222.24	<b>199.84</b>	<b>239.52</b>	<b>276.96</b>
		Quasi-3D (TSDT)	277.92	262.08	254.08	309.12	256.00	223.36	<b>200.64</b>	<b>240.64</b>	<b>278.24</b>